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ABSTRACT

This teacher resource manual has been developed to assist classroom teachers in implementing the Integrated Occupational Mathematics program for grades 8 and 9. The first chapter of this manual gives an introduction including resources and scope/sequence. The next two chapters describe the overviews and instructional strategies of the grade 8 and 9 themes. The themes developed at each grade level are classified as: managing your money; world of work; using math at home; and travel and recreation. The last chapter discusses generic strategies: (1) problem solving; (2) use of technology (including calculators and computers); (3) computational facility and estimation (dealing with the development of computational process, mental arithmetic skills, and estimation strategies); (4) using a math lab; and (5) evaluation. Resource materials are listed for each of the five topics. Lists 13 references. (YP)

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INTEGRATED OCCUPATIONAL PROGRAM

Teacher Resource Manual

ED313237

MATHEMATICS 8 AND 9

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Teacher Resource Manual

Mathematics 8 and 9

INTERIM – 1989

NOTE: This publication is a service document. The advice and direction offered is suggested only. Consult the Program of Studies/Curriculum Guide to identify the prescriptive contents of the Integrated Occupational Mathematics Program for Grades 8 and 9.

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INTRODUCTION

This *Teacher Resource Manual* has been developed to assist classroom teachers to implement the Integrated Occupational Mathematics 8 and 9 program. This manual provides:

- further information about the goals and objectives of the curriculum
- thematic contexts for the delivery of prescribed concepts, skills and attitudes
- suggestions for planning and implementing the program
  - instructional strategies
  - sequenced activities
  - a correlation of strategies and activities to learning resources
- suggestions for relating mathematical instruction to essential life skills and other applications across the curriculum
- suggestions for utilizing community resources in delivery of the mathematics program
- strategies for further developing student competence in using various methods of computation, in problem solving, and in the use of technology (i.e., the calculator and computer)
- suggestions for developing and using a mathematics lab
- suggestions for evaluating student progress.

Teachers are encouraged to use this manual as a practical planning and instructional tool. The binder format was chosen to enable teachers to add strategies, samples of student work and other activities that have proven effective through experience.

During cooperative planning sessions, pages may be easily removed, copied and shared with other Integrated Occupational Program teachers. This exchange will heighten teacher sensitivity to the content of other subject areas, and provide additional opportunities for students to relate mathematical competencies to applications in real life, the practical arts and other academic disciplines.

ORGANIZATION OF THE TEACHER RESOURCE MANUAL

This manual has been organized to provide ready access to both thematic units of instruction and generic strategies for developing essential concepts and skills in mathematics.

THEMES

Thematic units of instruction for Grade 8 and Grade 9 have been placed at the beginning of the manual. Each theme has been divided into two or more sub-themes. The chart below provides an overview of the themes and sub-themes developed at each grade level.

MANAGING YOUR MONEY		USING MATH AT HOME	
Sub-Themes		Sub-Themes	
Grade 8	Grade 9	Grade 8	Grade 9
• Banking • Money in the Marketplace • Managing Your Earnings	• Banking • Getting Value for Money Spent • Budgeting	• Calculating Food and Utility Costs • Using Measurement Skills • An Evening's Entertainment	• Making Home Improvements • Personal Nutrition and Growth
WORLD OF WORK		**TRAVEL AND RECREATION**	
Sub-Themes		Sub-Themes	
Grade 8	Grade 9	Grade 8	Grade 9
• Calculating Earnings • Measuring on the Job • Keeping Records	• Using Math on the Job • Working With Scale Drawings • Planning for Piecework	• Using Maps and Travel Schedules • Comparing Recreational Costs • Statistics in Sports	• Transportation • Planning a Trip • Sports and Fitness

GENERIC STRATEGIES

Generic strategies that may be effective in developing essential concepts and skills have been placed following the themes. These strategies can be used to provide focussed instruction at appropriate times within each theme. Components of instruction that are supported by the generic strategies provided in this manual include:

- Problem Solving
- Use of Technology
- Computational Facility and Estimation
- Using a Math Lab
- Evaluation.

Teachers are encouraged to reference these strategies as they plan activities within each theme of the program.

USE OF THE TEACHER RESOURCE MANUAL

It is recommended that THEMES provide the vehicle for instruction in Grade 8 and Grade 9. Activities suggested within each theme will direct teachers to GENERIC STRATEGIES that are useful in developing an understanding of number concepts, computational procedure and problem-solving process.

The following example illustrates how generic strategies have been integrated with themes developed in the manual.

- Turn to the first Grade 8 theme, **Managing Your Money**. Notice that within the "Rationale" for this theme, teachers are encouraged to reference generic strategies on:
  - Using a Math Lab
  - Computational Facility and Estimation
  - Problem Solving
  - Use of Technology.

- Locate the "Suggested Activities" for the first sub-theme, **Banking**. The following activities make reference to the use of generic strategies:
  (1) Encourage students to use their calculators when performing bank transactions. A review of calculator procedures may be worthwhile (see Use of Technology, "The Calculator").
  (2) Review money and decimal skills by playing "Money Dominoes" (see Using a Math Lab, "Number Systems and Operations").

- Locate the "Suggested Activities" for the second sub-theme, **Money in the Market Place**. These activities encourage teachers to reference additional generic strategies:
  (3) Discuss and model strategies for performing mental arithmetic and estimation (see Computational Facility and Estimation, "Developing Mental Arithmetic Skills" and "Developing Strategies for Estimation").
  (4) Provide students with a sheet of 10 by 10 grid paper. Use grid paper to determine fraction and percent equivalents. A blackline master for producing grid paper is provided in Using a Math Lab, "Ratio, Proportion and Percent".

- Locate the "Suggested Activities" for the third sub-theme, **Managing Your Earnings**. Again, suggestions are made regarding the use of generic strategies:
  (1) Help students to design a computer program that will determine earnings for given wage rates and hours of work (see Use of Technology, "The Computer").
  (4) Encourage students to use appropriate problem-solving strategies in developing personal budgets (see Problem Solving, "Using Strategies to Solve Problems").

Teachers are encouraged to examine each theme and its corresponding activities and learning resources well in advance of instruction. The activities and suggestions provided within each theme are numerous. **Advance planning should include a synthesis of effective strategies from the teacher's repertoire of personal experience together with suggestions in this manual considered most appropriate to student needs.**

LEARNING RESOURCES FOR MATHEMATICS 8 AND 9

STUDENT RESOURCES

BASIC LEARNING RESOURCES

The textbooks listed below meet the majority of the goals and objectives identified in this curriculum (authorization pending).

Pogue, Paul, et al. *Mathbase I: Essential Math Skills* (2nd Edition). Toronto, Ontario: Copp Clark Pitman, 1989.

Pogue, Paul, et al. *Mathbase II: Practical Skills and Applications* (2nd Edition). Toronto, Ontario: Copp Clark Pitman, publication anticipated in 1990.

Mathbase I: Essential Math Skills provides for the focussed development of concepts and skills identified in the Grade 8 and Grade 9 Program of Studies/Curriculum Guide. *Mathbase II: Practical Skills and Applications* will provide for thematic application of the prescribed concepts and skills in Grades 8 and 9. It is intended that appropriate sections of both *Mathbase I* and *Mathbase II* be used throughout the Grade 8 and Grade 9 programs. *Mathbase II* is scheduled for publication in the early spring of 1990.

TEACHER RESOURCES

RECOMMENDED LEARNING RESOURCES

Teacher resources designed to support instructional use of the basic learning resources include:

Pogue, Paul, et al. *Mathbase I: Essential Math Skills, Teacher's Edition* (2nd Edition). Toronto, Ontario: Copp Clark Pitman, publication anticipated in January, 1990.

Pogue, Paul, et al. *Mathbase II: Practical Skills and Applications, Teacher's Edition* (2nd Edition). Toronto, Ontario: Copp Clark Pitman, publication anticipated in 1990.

TECHNOLOGY AND MEDIA

COMPUTER COURSEWARE

The learning resources listed below have been approved by Alberta Education because they contribute significantly to the development of specific goals and objectives within this curriculum.

Title:	*Fast Facts*
By:	EduSoft, Berkeley, California, 1985
Components:	1 disk, guide (2 pages)
Objective:	To provide timed drills on whole number facts.

Title:	*Math Strategies: Problem Solving*
By:	Science Research Associates (Canada) Ltd., Willowdale, Ontario, 1985
Components:	2 disks, teacher's guide (28 pages), 20 student texts (108 pages)
Objective:	To provide instruction and practice in solving multiple-step problems using four problem-solving strategies: simplifying a problem, breaking a problem into parts, identifying needed additional information, and making a model of the problem.

An annotated list of additional courseware available in mathematics is provided in Alberta Education publications *Computer Courseware Evaluations: June 1985 to March 1986* (1986), *Computer Courseware Evaluations: January 1987 to December 1987, Volume VII* (1988) and *Computer Courseware Evaluations: January 1988 to December 1988, Volume VIII (1989)*.

REGIONAL RESOURCE LIBRARIES

Films and videos are avilable for loan through the five centres listed below. In some instances, computer software is also loaned. Catalogues of holdings are available upon request.

Zone I

Peace River Regional
c/o Peace River School District No. 10
P.O. Box 988
Peace River, Alberta
T0H 2X0
Telephone: 624-3187

Zone II and III

Central Alberta Media Service
c/o Sherwood Park Catholic School District
2017 Brentwood Boulevard
Sherwood Park, Alberta
T8A 0X2
Telephone: 464-5540

Zone IV

Alberta Central Regional Education Services
County of Lacombe No. 14
Bag Service 108
Lacombe, Alberta
T0C 1S0
Telephone: 782-6601

Zone V

South Central Alberta Film Federation
County of Wheatland No. 16
Box 90
Strathmore, Alberta
T0J 3H0
Telephone: 934-5028

Zone VI

Southern Alberta Regional Film Centre
c/o McNally School
P.O. Box 845
Lethbridge, Alberta
T1J 3Z8
Telephone: 320-7807

ACCESS NETWORK

ACCESS offers a variety of resources and services to teachers. For a nominal dubbing and tape fee, teachers may have ACCESS audio and video library tapes copied. ACCESS also offers a service called "Night Owl Dubbing". This allows educators to tape late night educational programs directly from their own televisions.

ACCESS publishes both an *Audio-Visual Catalogue* and a comprehensive schedule of programming, available on request.

For additional information, contact ACCESS NETWORK, Media Resource Centre, 295 Midpark Way SE, Calgary, Alberta, T2X 2A8 (from outside ofCalgary, telephone toll free, 1-800-352-8293; in Calgary, telephone 256-1100).

ADDITIONAL SUPPORT MATERIALS

The following learning resources have been identified as being potentially useful for the Integrated Occupational Mathematics program. None of these materials have been evaluated by Alberta Education and their listing is not to be construed as an explicit or implicit departmental approval for use. The list is provided as a service only to assist local jurisdictions in identifying potentially useful learning resources. The responsibility of evaluating these resources prior to selection rests with the local jurisdiction.

Baker, Trudy, et al. *Math You Need, Book 1*. Scarborough, Ontario: Nelson Canada, A Division of International Thomson Limited, 1982.

Hope, Jack, et al. *Mental Math in the Middle Grades*. Agincourt, Ontario: GLC/Silver Burdett Publishers, 1987.

Lane County Mathematics Project. *Problem Solving in Mathematics*. Agincourt, Ontario: GLC/Silver Burdett Publishers, 1983.

McFadden, Scott. *Math Warm-Ups For Junior High*. Agincourt, Ontario: GLC/Silver Burdett Publishers, 1983.

Mastering Essential Mathematics Skills. Toronto, Ontario: Doubleday Canada Ltd., 1981.

Teachers may also wish to enhance course presentation by using materials available from local media services, libraries and government agencies (e.g., pamphlets, films, video tapes, audio tapes, kits, picture sets, filmstrips).

SCOPE AND SEQUENCE

The Scope and Sequence chart provided on the following pages outlines the mathematical processes and skills that are developmentally addressed throughout Grades 8 and 9. In recognizing that students differ in the rate at which they acquire mathematical competencies, this chart is intended to assist teachers in:

- **assessing present levels of student performance**
- **diagnosing particular areas of skill deficiency**
- **sequencing instruction in a manner that will suit individual needs and growth patterns.**

In using the Scope and Sequence chart, it should be noted that:

- effort has been made to arrange process and skill in a linear sequence according to cognitive demand at each grade level
- the skills are developmental through Grades 8 and 9 (i.e., the spiral approach). **Students will reinforce and extend their understanding of skills developed in Grade 8 through their application in more sophisticated and complex settings at the Grade 9 level.**
- **the skills are interdependent and are not meant to be taught in isolation.** Although some skills may be mastered more effectively through discrete instruction, this approach is not advocated as a primary focus of instruction. The thematic structure permits a more holistic view of instruction through the linking of strategies and skills.

Teachers may also wish to examine the Scope and Sequence chart for the senior high school mathematics program (Mathematics 16 and 26). An understanding of the developmental progression of the process and skills occurring beyond Grade 9 will facilitate articulation between the junior and senior high school mathematics programs.

NOTE: The arrows indicate a spiral approach to skill development involving extension, enrichment, and/or reinforcement as appropriate.

PROBLEM SOLVING

GRADE 8	GRADE 9
Recognizes problem-solving situations at school, at home, and in the community where: • no readily apparent solution or means to the solution is evident • a person may be temporarily perplexed • there may be no answer, one answer or many answers • personal and societal factors may be involved as well as mathematical competencies	→
Demonstrates a desire to solve problems by: • asking questions/showing interest and curiosity • attempting to apply/transfer knowledge to problem situations • taking risks • displaying perseverance • using creative approaches/unconventional strategies • thinking critically/justifying strategies and solutions	→
Uses a variety of strategies to solve problems: • understands the problem – reads the problem several times – asks questions – identifies key words and their meanings – looks for patterns – identifies wanted, given, and needed information – identifies extraneous information – internalizes the problem by restating it in one's own words or by visualizing the problem – draws pictures/diagrams – uses concrete manipulatives	Uses a variety of strategies to solve problems: • understands the problem – interprets pictures/charts/graphs – simulates or models the problem situation – relates the problem to other problems previously encountered
• develops and carries out a plan – guesses and checks the result (thus improving the guess) – uses logic and reason – chooses and sequences the operations needed – sorts/classifies information – applies selected strategies – presents ideas clearly – selects appropriate calculating/measuring devices and methods – visualizes the problem – acts out or simulates the problem – applies patterns – estimates the answer – documents the process used – works with care – works in a group situation, sharing ideas – speaks to self with positive statements (e.g., "I can solve this.")	• develops and carries out a plan – uses a simpler problem (makes an analogy) – identifies factors relevant to the problem – collects and organizes data into diagrams, number lines, charts, tables, pictures, graphs or models – experiments through the use of manipulatives – breaks the problem down into smaller parts
• reviews and applies results – states an answer to the problem – restates the problem with the answer – explains the answer in oral/written form – determines if the answer is reasonable – discusses the process used with others – suggests other ways to solve the problem – checks the answer – considers the possibility of other answers/solutions	• reviews and applies results – makes and solves similar problems

	GRADE 8	GRADE 9
USE OF TECHNOLOGY	Develops an ability to effectively use the calculator. • identifies appropriate and inappropriate uses of the calculator • identifies and uses basic functions on the calculator (+, −, ×, ÷, =, decimal, clear) • clears and corrects entry errors • uses a calculator to add, subtract, multiply, and divide whole numbers and decimals • enters numbers in correct sequence for subtraction and division • determines whole number remainders for division • follows order of operations • selects from calculator display the number of decimal places appropriate to the context of a calculation • checks the reasonableness of answers obtained on the calculator	Develops an ability to use the calculator effectively • identifies and uses the percent function on the calculator • generates sets of multiples for a given number using the calculator
COMPUTATIONAL FACILITY AND ESTIMATION	Performs computations using paper-and-pencil algorithms within the parameters provided in this Scope and Sequence for whole numbers, decimals and fractions. Performs computations using a calculator with whole numbers and decimals (magnitude of numbers determined by the nature of the problem situation). Performs computations using mental arithmetic that are based on: • all single-digit operations • sequences of operations • doubling and halving • multiplying and dividing by powers of 10 • application of the commutative, associative and distributive properties • properties of zero Develops skills in estimation in order to determine: • the range of numbers within which a solution must lie • whether a solution in problem solving is reasonable • the reasonableness of computational results obtained using paper and pencil algorithms or the calculator Applies estimation strategies that include: • stating the largest and smallest reasonable answer to a problem before solving the problem • predicting whether a computation will result in a larger or smaller number • forecasting an order of magnitude for the result of computation (e.g., 10's, 100's, 1 000's) • predicting the magnitude of the result of a computation by rounding numbers to one significant digit Selects a method of computation (paper and pencil, calculator, mental arithmetic, estimation) that is appropriate to the nature of the problem, and provides reasons for the method chosen.	Performs computations using paper-and-pencil algorithms within the parameters provided in this Scope and Sequence for whole numbers, decimals, fractions and percent. Performs computations using a calculator with whole numbers, decimals, fractions and percent whose magnitude are determined by the nature of the problem situation. Performs computations using mental arithmetic with increased emphasis on the development of formal strategies. • compensation • computing from left to right → Applies estimation skills to the results of computation and problem solving, with increased emphasis on the development of formal strategies. • front-end estimation • clustering • rounding • compatible numbers →

NUMBER SYSTEMS AND OPERATIONS

WHOLE NUMBERS

GRADE 8	GRADE 9
Recognizes place value to one hundred thousands place.	
Reads and writes whole numbers to one hundred thousands place in context.	Uses a calculator to generate a set of multiples for a given number.
Compares/orders whole numbers to one hundred thousands place in applications.	Determines the lowest common multiple for pairs of numbers less than 10.
Rounds numbers to nearest 10, 100, 1000.	
Counts by multiples of 2, 3, 4, 5, 6, 10 and 12.	
Applies the associative, commutative and distributive properties to "mental exact" computations.	Determines pairs of factors related to basic multiplication facts up to 100.
Calculates/estimates sums and differences of numbers containing up to four digits (without the use of the calculator).	Determines prime factors up to 50.
Recalls products for numbers up to 10 x 10.	Expresses numbers up to 50 as the product of prime factors.
Determines "mental exact" products when multiplying by 10, 100 and 1000.	Determines the greatest common factor for pairs of numbers less than 50.
Recalls division facts for dividends up to 100.	
Recognizes different methods of representing division.	
Calculates/estimates products and quotients of numbers up to 3 digits by 1 digit (without the use of a calculator).	Calculates/estimates products and quotients of numbers up to 3 digits by 2 digits (without the use of a calculator).
Applies rules for the order of operations (brackets, multiplication, division, addition and subtraction).	
Applies whole number skills to problem-solving situations.	

DECIMALS

GRADE 8	GRADE 9
Recognizes place value to thousandths.	
Reads and writes decimals to thousandths in context.	
Compares/orders decimals to thousandths in applications.	
Rounds to nearest whole number, tenth and hundredth.	
Calculates/estimates sums and differences of numbers with 1 or 2 decimal places (without the use of a calculator).	
Calculates/estimates products of decimal numbers to thousandths, using 1-digit multipliers (without the use of a calculator).	Calculates/estimates products of decimal numbers to thousandths, using 1- or 2-digit multipliers (without the use of a calculator).
Calculates/estimates quotients for 2 decimal place numbers divided by 1-digit whole number divisors (without the use of a calculator).	Calculates/estimates quotients for 2 decimal place numbers divided by 1- or 2-digit whole number divisors (without the use of a calculator).
Determines "mental exact" products/quotients when multiplying or dividing decimals by 10 and 100.	Determines "mental exact" products/quotients when multiplying or dividing decimals by 10, 100 or 1000.
Applies decimal skills to problem-solving situations.	

NUMBER SYSTEMS AND OPERATIONS (continued)

	GRADE 8	GRADE 9
FRACTIONS	Illustrates the use of fractions in describing part of a whole, group, or point on a number line. Illustrates the relationship between whole numbers, decimals and fractions using a number line. Describes proper/improper fractions and mixed numbers through the use of objects, pictures and diagrams. Converts improper fractions to mixed numbers and vice versa. Compares and orders fractions in applications. Identifies and determines equivalent fractions (emphasis on 1/2, 1/3, 1/4, 1/5, 1/8 and 1/10). Recognizes and expresses fractions in basic form. Demonstrates addition and subtraction of proper fractions/mixed numbers with like denominators through the use of objects, pictures and diagrams. Writes number sentences to describe the addition and subtraction of fractions with like denominators. Applies fraction skills to problem-solving situations.	Relates fractions to division, converting fractions into decimal equivalents using a calculator. Recalls decimal equivalents for commonly used fractions (e.g., one-half, quarters, tenths). Determines common denominators for frequently used fractions (emphasis on 1/2, 1/3, 1/4, 1/5, 1/8 and 1/10). Demonstrates addition and subtraction for proper fractions/mixed numbers with unlike denominators through the use of concrete manipulatives. Writes number sentences to describe the addition and subtraction of fractions with unlike denominators. Demonstrates the multiplication and division of proper fractions/mixed numbers by whole numbers through the use of concrete manipulatives. Writes number sentences to describe the multiplication and division of fractions/mixed numbers by whole numbers.
INTEGERS		Recognizes the need for integers, and ways in which they are used. Uses vocabulary related to integers (e.g., positive, negative, plus, minus, above, below, gain, loss). Places integers on the number line. Compares and orders positive and negative numbers in applications. Demonstrates addition of pairs of integers between negative 25 and positive 25 through concrete manipulation/diagramatic representation. Writes number sentences to describe the addition of integers undertaken in the concrete mode. Applies integer skills to problem-solving situations.

	GRADE 8	GRADE 9
RATIO, PROPORTION AND PERCENT	Recognizes ratios as ordered pairs of numbers showing comparison of two quantities in the same unit. Uses concrete manipulation to construct ratios in the forms a:b, a as to b, and a/b. Generates equivalent ratios using single-digit whole number constants Verifies the equivalence of two ratios using common multiples or factors (e.g., $\frac{14}{6}\ \frac{(\div 2)}{(\div 2)} = \frac{7}{3}$) Recognizes proportions as statements about equivalent ratios. Describes practical problem situations by writing proportions. Determines the value of the missing component in a given proportion using the common factor/multiple method (e.g., $\frac{3}{4}\ \frac{(\times 25)}{(\times 25)} = \frac{?}{100}$) Demonstrates the concept of percent as ratio indicating parts out of 100. Applies skills in ratio, proportion and percent to problem-solving situations, using concrete manipulations/diagramatic representations	Recognizes "rates" as ratios showing comparison of two numbers with different units (e.g., 90km/2h, 3 items for $1.00). Describes practical problem situations involving rates by writing proportions. Determines the value of the missing component in a proportion involving rates using the common multiple/factor method. Converts whole number percents to ratios/decimals. Expresses ratios as percents and decimals (e.g., $\frac{a}{b} = \frac{?}{100}$, where b = 2, 4, 5, 10, 20, 25 or 50) Expresses 1- and 2-place decimals as percents (e.g., 0.5, 0.75, 0.4). Recalls fraction, decimal and percent equivalents for halves, quarters and tenths. Determines other fraction, decimal and percent equivalents through use of the calculator. Calculates/estimates a percent of a number in relevant applications.
GEOMETRY AND MEASUREMENT **GEOMETRY**	Identifies and distinguishes between horizontal, vertical, perpendicular, parallel and intersecting lines. Identifies/classifies/describes basic two-dimensional figures (rectangle, square, triangle, circle). Uses geometric tools (e.g., protractor, compass, straightedge, ruler, computer) to construct rectangles, squares, triangles and circles according to given specifications. Identifies and constructs models of basic three-dimensional figures (rectangular prism, cube, cylinder). Applies knowledge of geometric figures and relationships in practical situations.	Identifies and recalls characteristics of the parallelogram, hexagon and octagon. Uses geometric tools (e.g., protractor, compass, straightedge, ruler, computer) to construct the parallelogram, hexagon and octagon. Identifies and describes the relationship between the radius and diameter of a circle. Draws a circle, given either radius or diameter. Constructs geometric patterns/designs, using tools that may include the straightedge, compass, ruler, protractor, mira or computer.

	GRADE 8	GRADE 9
GEOMETRY AND MEASUREMENT (continued) **LENGTH**	Recognizes common metric units of length (mm, cm, m, km). Estimates and measures length, selecting metric units and tools appropriate to the situation. Draws lines according to given specifications, using metric units and tools. Converts measurements of length among commonly used metric units: – among mm, cm and m – between m and km. Illustrates the concept of perimeter, and explains its application to problem-solving situations. Estimates/measures/computes the perimeter of figures bounded by line segments.	→
AREA		Illustrates the concept of area, recognizing common metric units (cm^2, m^2) and their application in problem situations. Approximates the area of two-dimensional geometric figures using a square grid. Recognizes strategies/formulas for finding the area of rectangles and squares. Estimates/calculates the area of rectangles and squares, using units and strategies appropriate to the situation.
MASS	Describes mass, and recognizes common metric units (g, kg, t). Estimates and measures mass, selecting metric units and tools appropriate to the situation. Converts between g and kg, also between kg and t.	→
CAPACITY	Describes capacity, and recognizes common metric units (mL, L). Estimates and measures capacity, selecting metric units and tools appropriate to the situation. Converts between mL and L.	→
TIME	Uses a calendar, recognizing the relationship between days, weeks, months and years. Uses National Standards for numeric dating. Estimates/measures/records time on the 12-hour and 24-hour clock (using traditional and digital timepieces).	Converts between hours and minutes, and between minutes and seconds. Adds/subtracts hours and minutes in applications.
TEMPERATURE		Estimates and measures temperature on the Celsius scale. Recalls important temperatures on the Celsius scale (e.g., boiling/freezing points of water, normal body/room temperature). Determines temperature change, including changes from below zero to above zero.
ANGLE		Recognizes an angle and the degree as a unit of measure. Recognizes angles of 45°, 90°, 180° and 360°. Measures/draws angles from 0° to 180° using a protractor. Applies skills of angle measure in the construction of geometric figures/patterns/designs.

	GRADE 8	GRADE 9
DATA INTERPRETATION AND DISPLAY	Recognizes the use of statistics in real-life situations, and its effects on everyday activities. Reads and interprets information presented in list, table and chart form. Collects and records data using tally sheets and frequency tables. Uses tables and charts to group/sort numerical data and information according to specified criteria. Reads and interprets information presented in picture graphs, bar graphs and line graphs.	Interprets and determines arithmetical average in practical situations. Recognizes when and how to display data in the form of picture graphs, bar graphs and line graphs. Reads and interprets information presented in circle graphs. Recognizes how graphs may sometimes provide misleading information or distort the "true picture".
ALGEBRA	Distinguishes between the use of variables and constants in concrete situations. Uses variables to describe concrete situations (e g., number of coins in a jar). Uses variables to write mathematical expressions that describe practical situations (e.g., if the regular price of an item is reduced by five dollars, the sale price could be represented as R – 5). Evaluates mathematical expressions for given whole number values of the variable. Uses variables to write linear equations/formulas that describe practical situations (e.g., if each person at a party eats three hot dogs, the relationship between number of hot dogs and number of people can be described as H = 3 x P). Interprets formulas related to practical situations as word statements. Performs substitution into formulas in determining outcomes/solutions to routine problems.	Uses concrete manipulatives to demonstrate the concept of equality. Uses estimation and guess/check strategies to solve linear equations describing practical situations that have been written in any of the following forms: - x + a = b - ax = b - ax + b = c - x/a = b/c. Verifies solutions to linear equations by substitution.

I.O.P. MATHEMATICS 8
MANAGING YOUR MONEY

SUB-THEMES

- BANKING
- MONEY IN THE MARKET PLACE
- MANAGING YOUR EARNINGS

RATIONALE

Many students in junior high school are receiving some type of income, and need to develop strategies for managing the money they receive. This theme will enable students to develop:

- an understanding of banking procedures
- consumer skills as they relate to making a purchase in the marketplace
- strategies for monitoring income received through allowances, baby-sitting, etc.

Abundant opportunities exist for students to review basic computation and estimation skills. Thematic activities will permit teachers to diagnose individual student strengths and weaknesses, and to provide direct or focussed instruction as required in remediating student difficulties.

Cooperative planning with other teachers may provide ways of coordinating thematic activities in mathematics with topics studied in other subject areas. Such conferencing will ensure that instruction provided in mathematics will reinforce and enhance learning that is taking place within other disciplines. Many opportunities exist for experiential learning through community partnerships, classroom role-playing, and the use of manipulative materials.

Teachers are encouraged to reference the "Generic Strategies" section of this manual when planning for instruction. Strategies particularly relevant to the learning objectives and activities outlined in this theme can be found in:

- Using a Math Lab
- Computational Facility and Estimation
- Problem Solving
- Use of Technology.

THEMATIC OBJECTIVES

BANKING

- Recognizes major types of bank accounts:
  - chequing
  - savings.
- Maintains/verifies a current balance in a bank passbook.
- Completes a deposit and withdrawal slip.
- Reads and writes numbers necessary for filling out a cheque.
- Calculates the interest earned when given the amount and the principal.

MONEY IN THE MARKETPLACE

- Recognizes coin equivalents to $1.00 using pennies, nickels, dimes and quarters.
- Counts sums of money up to $50.
- Estimates total cost of a number of items.
- Calculates and estimates the amount of change to be received.
- Makes correct change for purchases when amount tendered is less than $50.
- Determines unit price.
- Determines the "best" deal by comparing unit prices.
- Recognizes percent as a ratio and uses it to calculate sale prices.

MANAGING YOUR EARNINGS

- Determines number of hours worked when given start and finish times.
- Calculates total earnings by multiplying hourly rate by the number of hours worked.
- Calculates annual earnings based on weekly/monthly incomes.
- Maintains a record of how personal money is spent.

CONTEXT FOR INSTRUCTION

PROBLEM SOLVING

- Selects the type of bank account most appropriate to personal goals and needs.
- Makes decisions on what is the "best" buy.
- Monitors personal income and plans simple budgets.

USE OF TECHNOLOGY

- Uses the calculator iteratively.
- Recognizes functions performed by a cash register.
- Reads and interprets computerized bank statements.
- Reads a time/job clock.

COMPUTATIONAL FACILITY AND ESTIMATION

- Uses basic computational algorithms.
- Uses mental arithmetic to determine exact values.
- Rounds to the nearest cent/dollar.
- Estimates sums and differences in consumer contexts.
- Uses estimation to check the reasonableness of calculations and solutions.
- Recognizes "per unit" as a form of division.

SUPPORTING CONCEPTS, SKILLS AND ATTITUDES

Number Systems and Operations	**Ratio, Proportion and Percent**	Geometry and Measurement	**Data Interpretation and Display**	**Algebra**

NUMBER SYSTEMS AND OPERATIONS

- Reads, writes and orders whole numbers and decimals.
- Identifies place value from 0.001 to 100 000.
- Rounds whole numbers and decimals as required.
- Performs basic operations with whole numbers and decimals.
- Uses fractions to represent parts of a whole.
- Recognizes/adds fractional parts of an hour.

RATIO, PROPORTION AND PERCENT

- Recognizes unit price as a ratio.
- Uses ratios to calculate unit price.
- Writes whole number percents as ratios and decimals.
- Calculates and estimates the percent of a number.
- Calculates sale price when given percent discount.

GEOMETRY AND MEASUREMENT

- Recognizes the relationship between days/weeks/months/years.
- Measures time on the 12-hour and 24-hour clock.
- Uses National Standards for numeric dating.

DATA INTERPRETATION AND DISPLAY

- Reads/interprets information presented in table/chart form (e.g., tables of principal, interest, and amount).
- Uses charts/tables to record personal spending.

ALGEBRA

- Uses the formula A = P + I.
- Develops/applies formulas for determining total earnings.

INTEGRATION ACTIVITIES

LANGUAGE ARTS

- Develops vocabulary useful in understanding and using banking services.
- Correctly spells numbers to 100 000 in context.
- Reads, interprets, and completes deposit and withdrawal slips.

SOCIAL STUDIES

- Recognizes the advantages/disadvantages of the use of various financial institutions and types of bank accounts.
- Recognizes the use of money as a form of trading/bartering.

PRACTICAL ARTS

- Counts money and makes change.
- Applies consumer skills to buying and selling.
- Calculates the amount of time spent on a job/project.

COMMUNITY PARTNERSHIP OPPORTUNITIES

- Invite guest speakers from local banks, credit unions, and loan companies to discuss services offered by these institutions:
  - features of major types of accounts
  - different interest rates available.
- Visit a local financial institution:
  - interview personnel regarding services offered
  - observe and use banking forms
  - examine the use of computers and other technologies.
- Visit a local supermarket. Encourage students to observe/determine/compare the unit prices of competing consumer products.
- Visit a local business where a job or time clock is in use to determine hours worked per day/per task.
- Invite the school business manager/secretary into the classroom to discuss/demonstrate appropriate methods of:
  - counting money
  - making change.

BANKING

BASIC RESOURCE CORRELATION

Mathbase 1

- Chapter 1: Understanding Numbers
  - About How Many
  - Numbers and Numerical Terms
  - Using [+] [−] [×] [÷] [=]
  - Your Calculator and Money
  - Using [C] and [CE] keys
- Chapter 3: Working With Numbers
  - Adding and Subtracting Money Amounts
- Chapter 5: Decimals and Measurement
  - Decimals and Money

COMPUTER SOFTWARE

Fast Facts

Math Strategies: Problem Solving

SUGGESTED ACTIVITIES

1. Encourage students to use their calculators when performing bank transactions. A review of calculator procedures may be worthwhile (see Use of Technology, "The Calculator").

2. Review money and decimal skills by playing "Money Dominoes" (see Using a Math Lab, "Number Systems and Operations").

3. Provide students with a copy of a bank passbook. Identify and discuss:

   - opening balance
   - withdrawals
   - deposits
   - interest earned
   - service charges
   - closing balance.

4. Simulate banking situations where students complete deposit/withdrawal slips and update bank passbooks. (A sample page of a bank passbook is provided as Resource 1: A Bank Passbook.)

5. Make a bulletin board display on "Banking". Include in the display:

   - copies of actual deposit/withdrawal slips
   - a sample passbook
   - a list of the various types of bank accounts
   - a list of services provided by the bank
   - a glossary of terms related to banking.

6. Plan a field trip to a local financial institution. Ask a staff member/manager to discuss/explain:

   - services offered by the bank
   - types of bank accounts
   - interest rates paid on different accounts
   - use of banking forms
   - use of technology in the banking business.

   This field trip could be planned in conjunction with the language arts and science classes. Students might write a letter of request and a thank-you letter as part of their language arts class. Applications of technology in banking, and its effects on our everyday lives could be discussed in science class.

MONEY IN THE MARKET PLACE

BASIC RESOURCE CORRELATION

Mathbase 1
- Chapter 2: Solving Problems
  - Look for a Pattern
- Chapter 8: Working With Percent
  - Meaning of Percent
  - Writing Percents as Decimals
  - Writing Decimals as Percents

Mathbase 2
- Chapter 5: Comparison Shopping

SUGGESTED ACTIVITIES

1. Ask students to "shop" for various items through the use of store catalogues. Given a specified amount of money, ask students to:

   - estimate the total cost of a list of items
   - determine if the total is less than the amount of money available
   - shop for the "best deal" on particular items
   - calculate the amount of change to be received from purchases made.

2. Provide each student or group of students with store catalogues/advertisements and a "shopping list". Ask students to:

   - note the availability and varying costs for different brands of each item
   - determine the unit cost for each item
   - determine the total cost of all items on the shopping list.

   This activity might take the form of a game. Divide the class into groups, and give each group of students the same shopping list and several different catalogues. The object of the game is to select items that will result in the lowest total cost for everything on the shopping list. (A table that might be used to record results is provided as Resource 2: The Better Buy.)

3. Encourage students to practise their mental arithmetic and estimation skills by simulating consumer situations in which these skills frequently arise.
e.g., If apples are 4 for $1.25, what will one apple cost?

Resource 3: Questions from the Supermarket, provides sample activities involving mental arithmetic and estimation skills.

Discuss and model strategies for performing mental arithmetic and estimation (see Computational Facility and Estimation, "Developing Mental Arithmetic Skills" and "Developing Strategies for Estimation").

4. Provide students with sheets of 10 by 10 grid paper. Using these grids, ask students to:

- shade $\frac{1}{10}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ of the grid.
- note the corresponding percentage of total area shaded
- make a table of fraction and percent equivalents for future reference.

A blackline master for producing 10 by 10 grid paper is provided in Using a Math Lab, "Ratio, Proportion and Percent".

5. Provide students with advertisements from stores where sales are described in a "percent-off" form. Ask students to calculate the discount and sale price. Students should determine discount using the common factor/multiple method when possible (see Using a Math Lab, "Ratio, Proportion and Percent"). The teacher may need to assist students in simplifying numbers.

Example 1:

Find the amount of discount and sale price on a pair of $50.00 slacks if the discount rate is 40%.

Discount Rate = $\frac{40}{100} \overset{(\div 2)}{=} \frac{20}{50}$

Sale Price = $50 – $20 = $30

Example 2:

Find the amount of discount and sale price on a pair of $50.00 slacks if the discount rate is 45%.
Discount = 0.45 × $50 = $22.50
Sale Price = $50 – $22.50 = $27.50

6. Ask students to design their own advertisements for items selected from a store catalogue. Student advertisements should include:

- a picture of the product
- the discount rate expressed as a percent
- the sale price.

MANAGING YOUR EARNINGS

BASIC RESOURCE CORRELATION

Mathbase 1

- Chapter 3: Working With Numbers
  - Units of Time
  - Adding Time
  - Working With A.M. and P.M.
- Chapter 4: Data Graphs
  - Organizing Data

SUGGESTED ACTIVITIES

1. Have students record the time spent on a task (e.g., homework, a hobby, a part-time job) for a one week period. Students should:

   - record starting time and finishing time each day in both 12-hour and 24-hour clock notation
   - calculate the total hours spent on the task throughout the week
   - calculate the money earned if paid minimum wage while engaged in this activity
   - convert the weekly wage to a monthly/yearly salary.

   Help students to design a computer program that will determine earnings for given wage rates and hours of work (see Use of Technology, "The Computer").

2. Provide students with a copy of a time card. Ask students to:

   - observe the abbreviations being used
   - distinguish between a.m. and p.m.
   - calculate total hours worked each day/week.

CLARIFICATION/EXAMPLE

WEEKLY TIME CARD

Name: John Smith

Employee # - 457

DESCRIPTION		CLOCK
Mon.	In	09:15
	Out	16:15
	Total	
Tues.	In	08:30
	Out	16:30
	Total	
Wed.	In	07:00
	Out	12:30
	Total	
Thurs.	In	
	Out	
	Total	
Fri.	In	12:15
	Out	21:30
	Total	
Sat.	In	09:00
	Out	17:30
	Total	
Sun.	In	
	Out	
	Total	

Checked by: _____ Week Total _____

a) Find the total number of hours John Smith worked each day.

b) John takes a one-hour lunch break each day for which he is not paid. Calculate his daily wage if he is earning $4.50/h.

c) Calculate the total number of hours John worked in the week. What would his weekly earnings be?

d) Complete John's card.

3. Ask students to keep a record of their personal spending for a period of several weeks. Classify expenditures into common categories (e.g., food, clothing, recreation/entertainment, travel). Encourage students to become aware of how they spend their money.

   Students may wish to use Resource 4: Record of Personal Spending, to record their expenditures.

4. Discuss the fact that a significant part of personal expenditure is used for pleasure rather than for absolute necessities. In the budgeting process, one should begin by planning for necessary expenditures (needs), and determine pleasure expenditures (wants) according to the balance.

   Encourage students to use appropriate problem-solving strategies in developing personal budgets (see Problem Solving, "Using Strategies to Solve Problems"). Ask students to identify:

   - their own needs and wants
   - variables and constants in the budgeting process
   - potential problems in their personal spending patterns
   - strategies for resolving these problems
   - alternative patterns for personal spending.

RESOURCE 1: A BANK PASSBOOK

DATE	*PARTICULARS	WITHDRAWAL	DEPOSIT	BALANCE

* PARTICULARS:
DEP = Deposit
WD = Withdrawal
IBB = InterBranch Banking
INT = Interest
SC = Service Charge
CHQ = Cheque

RESOURCE 2: THE BETTER BUY

Select different food items. Consult food advertisements in newspapers or visit grocery stores to obtain prices of various sizes and brands of the food items you have selected.

Complete a table like the one shown for each food item and determine which is the best buy for that particular item.

FOOD ITEM: _____

STORE	BRAND	SIZE	COST	UNIT PRICE

FOOD ITEM: _____

STORE	BRAND	SIZE	COST	UNIT PRICE

FOOD ITEM: _____

STORE	BRAND	SIZE	COST	UNIT PRICE

The best buy is _____ because _____

_____.

Factors other than price that I should consider in buying this item are: _____

_____.

RESOURCE 3: QUESTIONS FROM THE SUPERMARKET

Use mental arithmetic and/or approximation to find exact or nearly-exact answers to the following questions on supermarket prices. Check the answers to questions you are unsure about with a calculator.

1. The sale price for lettuce is two heads for $1.59. How much will one head cost?

2. The sale price for canned peas is two cans for $1.09. How much will one can cost?

3. a. Small cans of soda are on sale at eight cans for $3.00. How much will one can cost?

   b. If the eight cans of soda were purchased one at a time, what would be the total cost?

4. The sale price for light bulbs is three for $2.30. How much will one light bulb cost?

5. If a coupon says "9¢ off any size of Good Home Salad Dressing", how much will a bottle that regularly sells for $1.89 cost?

6. A store coupon says "Save 13¢ on your next purchase of ABC Spray Cleaner". If a bottle of the cleaner regularly costs $1.09, what is the sale price?

7. a. What is the sale price of a giant-sized bag of Big Boy dog food if the store coupon is for 23¢ off and the regular price is $5.99?

   b. The giant-sized bag of XYZ dog food regularly sells for $5.71. It is not on sale. If both brands are equally good, which is the better buy, Big Boy dog food on sale or XYZ at the regular price?

8. The label on a bottle of hair conditioner states "Mail enclosed coupon and receive a $.50 refund". If a bottle of conditioner regularly sells for $2.98, what will the price of the conditioner be when the coupon is used? (Remember to include current postal rate.)

9. The label on a bottle of hair conditioner states "Mail enclosed coupon and receive a $.75 refund". If the conditioner regularly sells for $2.25, what will the price be when the coupon is used? (Remember to include current postal rate.)

10. A giant-sized box of laundry detergent has these words on it: "Price marked is 25¢ off regular price". The price on the box is $7.22. What is the regular price?

RESOURCE 4: RECORD OF PERSONAL SPENDING

DAY	CATEGORY	AMOUNT
Sunday		
Monday		
Tuesday		
Wednesday		
Thursday		
Friday		
Saturday		

I.O.P. MATHEMATICS 8
WORLD OF WORK

SUB-THEMES

- CALCULATING EARNINGS
- MEASURING ON THE JOB
- KEEPING RECORDS

RATIONALE

This theme provides students with opportunities to:

- develop skills required in calculating wages
- apply measurement skills in a variety of situations relevant to the workplace
- develop strategies for recording time and materials required to complete various work-related tasks.

Relating activities to part-time jobs students may presently have or to employment opportunities currently available within the local community will ensure that learning experiences are relevant and meaningful.

Students will continue to develop and reinforce computational skills involving the use of whole numbers, decimals, fractions and percentages. Calculators should be used on a regular basis in performing many of the routine calculations (e.g., wage calculations, quantity and cost of materials required for a job). Measurement activities should place emphasis on estimation before application of actual skills in measurement. This approach will enable students to establish a "feel" for the size of various units, and enhance ability to select units that are appropriate to the task or problem.

Cooperative planning with other subject areas will facilitate a broader and more meaningful coverage of thematic objectives. The community offers additional resources for providing real life experience with many of the learning objectives.

Teachers are encouraged to reference the "Generic Strategies" provided in this manual when planning for instruction. Strategies particularly relevant to the learning objectives and activities outlined in this theme may be found in:

- Problem Solving
- Use of Technology
- Computational Facility and Estimation
- Using a Math Lab.

THEMATIC OBJECTIVES

CALCULATING EARNINGS

- Uses a time card in order to determine number of hours worked.
- Determines gross pay when given the hourly rate of pay and number of hours worked.
- Calculates annual earnings based on weekly/monthly incomes, and vice versa.
- Recognizes compulsory deductions from earnings:
  - Income Tax
  - Canada Pension
  - Unemployment Insurance.
- Calculates net earnings (i.e., take-home pay) given gross earnings and deductions.

MEASURING ON THE JOB

- Applies skills of estimation and measurement to work-related situations:
  - selects units and tools appropriate to the situation
  - estimates and measures length, mass, capacity and time
  - converts among commonly used units as required.
- Applies perimeter concepts in work-related situations.
- Reads meters and gauges that are used for measurement in work-related situations.
- Adds, subtracts, multiplies and divides measurements as required in work-related situations.

KEEPING RECORDS

- Reads/interprets time measured on a 12-hour and 24-hour clock.
- Determines number of work days in a month/year by using a calendar.
- Estimates/calculates the quantity and cost of materials used in a project.
- Estimates the total cost of a project, including labour and materials
- Completes work orders/invoices, using National Standards for numeric dating.

CONTEXT FOR INSTRUCTION

PROBLEM SOLVING

- Investigates and compares the pay for different jobs, using hourly/weekly/monthly rates of pay.
- Solves problems that require the application of measurement skills to a variety of work-related situations.
- Uses problem-solving strategies in determining the quantity/cost of materials required for a project.

USE OF TECHNOLOGY

- Uses a calculator iteratively.
- Recognizes the use of a time clock/job clock.
- Uses computer programs to develop/reinforce skills in problem solving, computation and measurement.
- Uses appropriate measurement tools/meters/gauges in work-related situations.

COMPUTATIONAL FACILITY AND ESTIMATION

- Reviews/maintains basic computational algorithms.
- Performs routine calculations with the calculator on a regular basis.
- Uses estimation to check computational results that are related to earnings and quantity/cost of materials used in projects.
- Uses mental arithmetic where appropriate to expedite solutions to quantitative problems.

SUPPORTING CONCEPTS, SKILLS AND ATTITUDES

Number Systems and Operations	Ratio, Proportion and Percent	Geometry and Measurement	Data Interpretation and Display	Algebra

NUMBER SYSTEMS AND OPERATIONS

- Rounds whole numbers and decimals as required.
- Performs computations using mental arithmetic/paper-and-pencil algorithms/calculator.
- Rounds amounts of money to the nearest cent/dollar.
- Reads numbers represented on various scales and measuring devices.
- Determines "mental exact" products when multiplying or dividing by 10, 100 and 1000.
- Applies rules for the order of operations.

RATIO, PROPORTION AND PERCENT

- Uses ratio to represent the comparison of two quantities.
- Recognizes proportions as statements about equivalent ratios.
- Describes practical problem situations by writing proportions.
- Determines the value of the missing component in a proportion using the common factor/multiple method.

GEOMETRY AND MEASUREMENT

- Identifies/constructs squares, rectangles, triangles, and circles.
- Recognizes common metric units of:
  - length (mm, cm, m, km)
  - capacity (mL, L)
  - mass (g, kg)
- Estimates and measures length/capacity/mass, selecting metric units and tools appropriate to the situation.
- Converts measurements as required among commonly used units.
- Estimates and measures perimeter.
- Estimates/measures/records time on the 12-hour and 24-hour clock.

DATA INTERPRETATION AND DISPLAY

- Reads and interprets information presented in chart/table/graph form.
- Collects and records data in chart/table form.
- Makes inferences based on statistical data.

ALGEBRA

- Uses a formula to find perimeter.
- Uses variables to represent the relationship among gross pay/deductions/take-home pay.
- Generates formulas for converting among units of measurement (e.g., 5L = ? mL).

INTEGRATION ACTIVITIES

LANGUAGE ARTS

- Uses reporting and note-taking skills in completing invoices and work orders.
- Uses research skills when investigating/comparing the pay for different types of jobs.
- Writes a letter of application for part-time employment.

SCIENCE

- Estimates/measures length, mass, capacity and time when collecting data for scientific investigations.
- Compares measuring devices used in the science laboratory (e.g., graduated cylinders, balance scales) with those used in the workplace.

SOCIAL STUDIES

- Recognizes/infers the relationship between compulsory deductions from earnings and the social system.
- Investigates the history of income tax.
- Relates employment statistics and trends to our changing social structure.

PRACTICAL ARTS

- Estimates/measures length, mass, capacity and time in work-related situations.
- Uses appropriate measuring devices/meters/gauges.
- Develops clerical skills that are related to project work in the practical arts classroom (e.g., monitors time spent on project, determines quantity/cost of materials used in a project, completes invoices and work orders).
- Interprets tables and charts in order to complete a procedure or task.

COMMUNITY PARTNERSHIP OPPORTUNITIES

- Invite a representative from local business/industry to discuss:
  - the use of geometry and measurement in various professions/trades (e.g., carpenter, painter, garment or drapery fabricator, food service operator)
  - employer expectations, salary ranges, compulsory deductions and pay methods.
- Visit local business and industry in order to observe:
  - how hours worked by employees are monitored/recorded
  - how gross pay, deductions and net pay are determined
  - the application of measurement skills
  - meters, gauges and other technologies used in measurement.
- Estimate the materials required in repairing or renovating various community facilities:
  - painting a building
  - fencing a yard or park.
- Invite representatives from local government agencies to discuss employment opportunities, legal responsibilities of employer/employee, and compulsory deductions from pay:
  - Canada Manpower
  - Labour and Employment Standards
  - Unemployment Insurance Commission
  - Revenue Canada
- Investigate employment skills and work expectations through job shadowing programs.

CALCULATING EARNINGS

BASIC RESOURCE CORRELATION

Mathbase 1
- Chapter 3: Working With Numbers
  - Multiplying by a 2-Digit Number
  - Multiplying a Money Amount by a Whole Number
  - Dividing a Money Amount by a Whole Number

Mathbase 2
- Chapter 1: Earning a Living

COMPUTER SOFTWARE

Fast Facts

Math Strategies: Problem Solving

SUGGESTED ACTIVITIES

1. Provide opportunities for students to develop skill in recording 24-hour time and determining hours worked (i.e., calculating the interval between a pair of times). Encourage students to keep a time card of "class working hours" throughout the duration of the theme. A sample time card that might be used for this purpose is provided as Resource 1: Sample Time Card.

   In using the time card, students might be asked to:

   - identify abbreviations used
   - convert 24-hour time to 12-hour time
   - calculate the number of hours and minutes worked each day
   - determine total hours worked each week
   - calculate gross pay based on the minimum wage.

2. Simulate a work situation by asking students to use information on their time card of "class working hours" to determine gross pay they might receive if paid the minimum wage. Students might be told to include homework as evening time worked, and to pay themselves "time-and-a-half" for any homework time worked over five hours each week. Encourage students to discuss and compare their weekly time cards and "gross pay".

3. Using the local newspaper, ask each student to find three employment opportunities where wages are paid on an hourly basis. Compare the wage rates offered for each of the jobs, and discuss reasons why the remuneration varies:
   e.g., - type of work
   - education required
   - hours of work
   - experience required.

4. Ask students (individually or in small groups) to identify five jobs they feel they might be able to perform (e.g., babysitting, mowing lawns, working at a fast-food outlet). Determine the hourly rate of pay that might be reasonably expected for each of the jobs identified. Summarize results in a table or chart, indicating:

   - the type of work
   - the hourly rate
   - the gross pay for five hours of work.

5. Discuss the difference between a "salary" and a "wage". Find employment opportunities in the newspaper that quote remuneration as an hourly rate, and by the day, week, month and year. Compare the remuneration offered for each of these jobs by computing equivalent hourly, daily, weekly, monthly and yearly rates of pay. A table that might be used to summarize the results of this investigation is provided in Resource 2: Comparing Rates of Pay.

6. Students might be given additional practice in determining income from employment by changing wages and salaries into yearly incomes. If the pay is an hourly rate, assume a 40-hour week for 52 weeks a year.
   e.g., – $108 per week
   – $359 per month
   – $4.80 per hour
   – $457 every two weeks.

   Review time relationships (i.e., hours, days, weeks, months, year) as required. Computation should be done using a calculator, with emphasis placed on the use of estimation in checking the reasonableness of results (see Computational Facility and Estimation, "Developing Strategies for Estimation" and Use of Technology, "The Calculator").

7. Examine a variety of actual "statements of earnings" (i.e., pay stubs). Identify the areas on each statement that indicate:

   - gross pay
   - deductions
   - net pay.

   Discuss the kinds of "compulsory" deductions that every employee must pay, and "voluntary" deductions that some employees may pay.

8. Ask students to investigate the compulsory deductions (e.g., Income Tax, Canada Pension, Unemployment Insurance) for several different incomes. Deductions may be determined through the use of appropriate tables and charts. Once deductions have been determined, ask students to calculate the net pay for each income.

   Summarize and record the results of this activity using Resource 3: Statement of Earnings.

MEASURING ON THE JOB

BASIC RESOURCE CORRELATION

Mathbase 1

- Chapter 3: Working With Numbers
  - Selecting the Best Unit
- Chapter 5: Decimals and Measurement
  - Multiplying and Dividing by 10, 100, 1000
  - Linear Units of Measure: Conversions
- Chapter 6: Working With Decimals
  - Mass
  - Selecting the Best Unit

SUGGESTED ACTIVITIES

1. Provide opportunities for frequent hands-on experience in estimating and measuring length, capacity and mass. A variety of strategies for developing skills of estimation and measurement are provided in the generic strategies section of this manual (see Using a Math Lab, "Geometry and Measurement").

   The suggestions which follow may also be useful in developing estimation and measurement skills.

   - Use "mind pictures" or referents to facilitate estimation with units of length, capacity and mass.
   - Have students make their own metric tapes.
   - Ask students to estimate and measure:
     - the length of a room in metres
     - the width of a table in centimetres
     - the height of a window in centimetres
     - the width of a doorway in centimetres.
   - Plan a walking rally or treasure hunt using given distances.
   - Display containers of various sizes and shapes. Ask students to estimate and then measure the capacity of these containers.
   - Display various consumer products obtained from the hardware store, lumber yard or supermarket. Ask students to estimate the mass of these products, and then measure mass as a check against estimates that have been made.

2. Discuss work-related situations in which estimates of measure are required, and compare these situations to others where exact measurements are required. Invite a carpenter to discuss the use of estimation and exact measurement in the building industry.

   Model the use of various strategies for making estimates of measure:

   - using a referent
   - chunking
   - unitizing

   (see Using a Math Lab, "Geometry and Measurement").

3. Develop the concept of perimeter through the use of geoboards/dot paper/centimetre grid paper (see Using a Math Lab, "Geometry and Measurement"). A blackline master for producing centimetre grid paper is provided in the Grade 8 theme "Using Math at Home".

   Once students recognize that perimeter means "distance around", provide opportunity for students to:

   - determine the perimeter of familiar objects in the room
   - determine the perimeter of objects represented by diagrams/scale drawings
   - determine the quantity/cost of materials required for various work-related projects that involve perimeter:
     e.g., – moulding for a room
     – trim for a wood project/sewing project
     – fencing material for a garden.

   Encourage students to devise their own formulas for finding the perimeter of rectangles, squares and triangles.

4. Encourage students to recognize the importance of measurement in a variety of work-related situations. Make a collage of pictures collected from newspapers and magazines that depict the use of:
   - length
   - mass
   - capacity
   - perimeter
   - time
   - temperature.

   Discuss appropriate units and tools of measurement that might be used in each of the situations illustrated.

5. Converting between metric units of measure provides opportunity for review of multiplication and division by multiples of ten. Ask questions that will encourage students to "think through" these processes and develop strategies of their own. Rules should be applied <u>after</u> the processes are understood.

   Example: 4500 cm = _____ m
   - Will I have more or fewer metres than centimetres?
   - If I need fewer metres, what operation must I use?
   - How do I determine the number by which I must divide?
   - In what direction will I move the decimal, and how many places do I move it? Why?
   - Does the answer seem reasonable?

   Possible strategies that might be used for converting among units of measure include:

   - writing a formula
   - making a chart
   - using a calculator algorithm.

6 Provide students with experience in reading the scales on a variety of measurement gauges/meters used in work-related situations. Borrow gauges, meters and other tools of measurement used by students in their practical arts and science courses.

Assist students to read these scales by developing skills of interpolation. Examine several different scales on the overhead and coach students in determining intermediate values. A variety of scales that require interpolation are provided in the Grade 8 theme "Using Math at Home".

7. Examine technical drawings/patterns that are being used by students in their practical arts courses. (Conferencing with practical arts teachers will facilitate the identification of technical drawings and patterns used by students in work-related situations.)

Assist students to interpret these drawings/patterns, and to perform calculations that are required in:

- determining the quantity of materials required for the project
- constructing/completing the project.

8. Provide frequent puzzles involving estimation and measurement that require the application of problem-solving strategies (see Problem Solving: "Using Strategies to Solve Problems").

CLARIFICATION/EXAMPLE

WATER PROBLEM[1]

You are given a 3 L container and a 5 L container. How can you use these containers to measure out exactly 1 L of water? Is it possible to measure out exactly 1 L of water using a 4 L container and a 6 L container?

Solution:
Fill the 3 L container with water and transfer its contents to the 5 L container. Refill the 3 L container and pour enough water into the 5 L container to fill it. There will then be 1 L of water left in the 3 L container. The second question cannot be done.

Teacher Note:
This problem may lead to a discussion of the properties of addition and subtraction for even and odd numbers.

[1] Alberta Education. *Let Problem Solving Be The Focus In The 1980's*, p. 113, 1983.

CLARIFICATION/EXAMPLE

SCALE PROBLEM[1]

How many nails will balance one cube?

– block

– bolt

– nails

Answer: 3

A variety of books containing recreational puzzles of this nature can be obtained from local bookstores and libraries.

KEEPING RECORDS

BASIC RESOURCE CORRELATION

Mathbase 1

- Chapter 1: Understanding Numbers
  - Coding: Numeric Dates
  - The Twenty-Four Hour Clock
- Chapter 2: Solving Problems
  - Using a Table or an Array

SUGGESTED ACTIVITIES

1. Provide opportunities for students to read/interpret time on both the 12-hour and 24-hour clock using traditional and digital time pieces. Activities should enable students to:

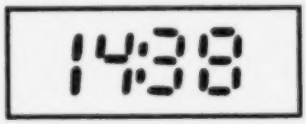
   - read/record time using both 12-hour and 24-hour notation
   - distinguish between A.M. and P.M. when using 24-hour notation
   - convert between 12-hour and 24-hour notation.

[1] Alberta Education. *Let Problem Solving Be The Focus In The 1980's*, p. 113, 1983.

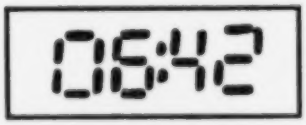
CLARIFICATION/EXAMPLE

a. Record this time using 12-hour and 24-hour notation.

b. How would you write "12 o'clock noon" in 24-hour notation?



c. Record these times using 12-hour notation. Be sure to indicate a.m. or p.m.



d. School starts at 8 a.m.. Are you late? How many minutes are there before school starts?

2. Ask students to identify various instances in real life where they must read/interpret time expressed in 24-hour notation. Applications may include:

   - travel schedules
   - hospital time
   - traffic/parking signs
   - job clocks used in business/industry.

3. Provide students with a copy of a calendar for the current year. Ask students to:

   - identify the normal work days/major holidays within each month
   - determine which month has the greatest number/least number of working days
   - prepare a chart indicating the total number of working days/holidays within each month.

4. Ask each student to select/design a simple project involving the use of measurement skills. Projects may relate to a practical arts course taken by the student (e.g., a sewing, wood or cooking project). Provide time for students to plan and complete their project, using the mathematics classroom, other facilities in the school, and/or facilities available at home. In completing their project, each student should:

   - determine the quantity/cost of all materials used in their project
   - monitor and record the amount of time spent in planning and completing the project
   - describe the measurement skills used in completing the project
   - determine an appropriate selling price for the completed project, considering cost of materials and labour.

   Ask students to present their completed projects to the class, and describe the documentation process used while completing the project.

   Evaluation of the project should be based on the student's documentation of materials/time/cost, rather than on the quality of the finished product. Teachers may wish to use an interview guide in evaluating each student's efforts (see Evaluation: Resource 4: Interview Guide for Project Work).

5. Provide students with actual copies of work orders and invoices used in business and industry. (Teachers might obtain actual copies of work orders and invoices used by students in their practical arts courses.) Discuss the purpose of these forms, and simulate situations that require the student to complete work orders/invoices (see Resource 4: Using Work Orders).

   Encourage students to use calculators in performing necessary computations, and to always check the reasonableness of the results they obtain through the use of estimation skills (see Use of Technology, "The Calculator" and Computational Facility and Estimation, "Developing Strategies for Estimation").

RESOURCE 1: SAMPLE TIME CARD

TIME CARD

EMPLOYEE: *Jane Doe*

Social Insurance Number: *623 189 458*

Week Ending: *October 27*

	MORNING		AFTERNOON		EVENING		HOURS
	IN	OUT	IN	OUT	IN	OUT	
M	9:00	12:00	13:00	17:30			
T	9:00	11:30	12:00	17:00			
W	8:00	11:00	12:00	16:30			
Th			12:00	17:00	18:00	20:30	
F			12:30	16:30	17:30	21:00	
S							

	HOURS	RATE	EARNINGS	
REGULAR				
OVERTIME				
TOTALS				

RESOURCE 2: COMPARING RATES OF PAY

Identify different employment opportunities that pay on an hourly, daily, weekly, monthly, yearly basis. Compare the remuneration offered for each of these jobs by computing equivalent hourly, daily, weekly, monthly and yearly rates of pay. Assume there are 20 workdays in each month.

TYPE OF JOB AND QUOTED RATE	RATE OF PAY				
	HOURLY	DAILY	WEEKLY	MONTHLY	YEARLY
Hourly:					
Daily:					
Weekly:					
Monthly:					
Yearly:					

QUESTIONS:

1. Which job pays the highest salary?

   _____

2. Which job pays the lowest salary?

   _____

3. Give some reasons why these jobs have different salary scales.

   _____

   _____

   _____

   _____

4. Would you prefer your salary to be based on an hourly, daily, weekly, monthly or yearly rate? Why?

   _____

   _____

RESOURCE 3: STATEMENT OF EARNINGS

Example One:

Employee	Net Claim Code	Gross Pay	U.I.C.	C.P.P.	Income Tax
Deductions					
Medical Insurance	Union Dues	Dental Plan	Other	Total Deductions	Net Pay

Example Two:

STATEMENT OF EARNINGS				
Basic Pay	Overtime Pay		Additional Pay	Gross Pay
Voluntary Deductions				Deductions
Compulsory Deductions	Canada Pension	Unemployment Insurance	Income Tax	Deductions
Date of Issue	Employee			Net Pay
Pay Period	Employee #	Dept. #	Employer	

RESOURCE 4: USING WORK ORDERS

Work orders are used in service stations and other kinds of repair shops to keep a record of work done for a customer.

Janet Smith of 1988 – 86 Street, Somewhere, Alberta is having a muffler system replaced and an oil change on her 1989 Camaro Convertible. She left the car at 8:00 A.M. and can be reached at 241-6666 when the job is complete. She has been told her car will be ready by noon the next day .

Complete a work order for Janet Smith that is based upon the information provided below. Use the work order form provided on the following page.

Labour Costs

a. muffler and pipes replacement		$45.75
b. exhaust pipe		6.00

Parts

a. muffler		$38.95
b. exhaust pipe		15.95
c. tail pipe		13.85
d. clamps	2@	3.25

Gas, Oil, Grease

a. 10W30 oil	5L@	$ 2.25
b. grease		7.75

I HEREBY AUTHORIZE ALL REPAIR WORK AS DESCRIBED AND ALL NECESSARY REPLACEMENT OF PARTS. _____	CUSTOMER		PHONE	OFFICE USE ONLY
	ADDRESS		CALL WHEN READY AM PM	
PARTS	MAKE	MODEL	ODOMETER	
	TIME RECEIVED	TIME PROMISED	WRITTEN BY	
	LICENSE NO.	SERIAL NO.	DATE	

		JOB DESCRIPTION	LABOUR

GAS, OIL, GREASE			TOTAL LABOUR	
	ALL ALTA SERVICE		TOTAL PARTS	
	1986 – 88 Street		GAS, OIL, GREASE	
	Somewhere, Alberta		OTHER	
Total	T4A 5W8		TOTAL AMOUNT	

I.O.P. MATHEMATICS 8
USING MATH AT HOME

SUB-THEMES

- CALCULATING FOOD AND UTILITY COSTS
- USING MEASUREMENT SKILLS
- AN EVENING'S ENTERTAINMENT

RATIONALE

Opportunities are provided for students to recognize instances where math is used in their home, and to develop a variety of skills frequently used in performing household tasks. Many of the activities will enable students to reinforce previously developed skills by applying them to familiar household situations.

Measurement must be viewed as a strategy used to gather information required in problem-solving situations, and not as an activity in itself. Throughout this theme, measurement activities should be related to real life tasks with emphasis on the use of estimation before application of actual skills in measurement. This approach will enable students to get a "feel" for the size of various units and will enhance their ability to select units that are appropriate to the task or problem.

Many opportunities exist for the integration of learning tasks with other subject areas. Cooperative planning with teachers in other subject areas will identify relevant and practical learning activities. The nature of this theme makes each student's home a community partnership, and each care-giver a community resource person. The teacher should be sure to take advantage of this in planning thematic activities.

Teachers are encouraged to reference the "Generic Strategies" section of this manual when planning for instruction. Strategies particularly relevant to the learning objectives and activities outlined in this theme can be found in:

- Problem Solving
- Use of Technology
- Computational Facility and Estimation
- Using a Math Lab.

THEMATIC OBJECTIVES

CALCULATING FOOD AND UTILITY COSTS

- Estimates/calculates total monthly food costs.
- Estimates the cost of a simple recipe given the unit price of each ingredient.
- Compares the cost of dining out to eating at home.
- Calculates unit price.
- Determines the "best deal" by comparing unit prices.
- Reads and interprets home costs displayed in bar/line/picture graphs.
- Reads and interprets scales/calibrations on water and natural gas meters
- Calculates utility costs that are based on utility meter readings.
- Keeps a record of home costs and displays information in chart form.

USING MEASUREMENT SKILLS

- Estimates/measures length in mm, cm, m, km.
- Determines perimeter in practical situations through the use of estimation, measurement and calculation.
- Determines the cost of materials required for simple home projects.
- Estimates/measures the mass of household items in g and kg.
- Estimates/measures liquid capacity in mL and L.
- Uses appropriate measurement strategies in following a simple recipe.

AN EVENING'S ENTERTAINMENT

- Reads and interprets a television/radio schedule.
- Estimates/calculates the cost of watching television (e.g., electricity, cable charges, cost of pay channels).
- Compares the cost of renting a video with going to the movie theatre.
- Recognizes the need for mental arithmetic in games (e.g., cribbage, monopoly).
- Performs mental arithmetic in a game situations.
- Creates geometric designs using a variety of construction instruments and techniques.

CONTEXT FOR INSTRUCTION

PROBLEM SOLVING

- Uses a problem-solving strategy for comparing the cost of consumer items and determining the "best" buy.
- Solves problems that require the application of measurement skills in real life situations.
- Develops formulas for finding the perimeter of two-dimensional figures.

USE OF TECHNOLOGY

- Uses the calculator in determining the quantity and cost of food goods/utilities/construction materials.
- Uses the calculator and appropriate formulas to determine perimeter.
- Reads meters and gauges used in the home.
- Uses computer programs to maintain/reinforce skills in geometry/measurement.
- Uses a variety of measurement tools.

COMPUTATIONAL FACILITY AND ESTIMATION

- Estimates length, perimeter, mass, and capacity as required in practical situations.
- Estimates the total cost of a project.
- Performs mental arithmetic in practical situations.
- Computes using both paper-and-pencil and the calculator in problem situations.

SUPPORTING CONCEPTS, SKILLS AND ATTITUDES

Number Systems and Operations	Ratio, Proportion and Percent	**Geometry and Measurement**	**Data Interpretation and Display**	**Algebra**

NUMBER SYSTEMS AND OPERATIONS

- Rounds whole numbers and decimals.
- Reads/writes whole numbers, decimals and fractions in context.
- Uses the properties of whole numbers in performing computations.
- Performs arithmetical operations using mental arithmetic/pencil-and-paper algorithms/calculator.

RATIO, PROPORTION AND PERCENT

- Determines the missing element of a proportion.
- Uses ratio/proportion to find unit price.
- Uses ratio/proportion/percent to represent monthly costs.

GEOMETRY AND MEASUREMENT

- Identifies/constructs basic two-dimensional figures.
- Recognizes metric units of:
  - length (cm, m, km)
  - capacity (mL, L)
  - mass (g, kg)
- Estimates and measures length/capacity/mass, selects units and tools appropriate to the situation.
- Estimates, measures and computes perimeter.
- Uses geometric tools to construct patterns and designs.

DATA INTERPRETATION AND DISPLAY

- Uses tables and charts to sort/organize numerical data.
- Interprets information displayed in chart/table form.
- Reads and interprets bar/line/picture graphs.

ALGEBRA

- Uses a formula to determine perimeter.
- Interprets formulas that are used to determine utility costs.

INTEGRATION ACTIVITIES

LANGUAGE ARTS

- Discusses the pros and cons of watching television.
- Relates the intrinsic value of various leisure activities to their costs.

SOCIAL STUDIES

- Relates food/utility costs to economics/inflation.
- Recognizes how the "household dollar" is spent.

SCIENCE

Science students use measurement skills in collecting data related to various investigations that are undertaken. Conferencing and cooperative planning with science teachers will identify:

- units of measure most frequently used in science class
- measurement tools and gauges that the student must be able to use
- relevant contexts in which measurement is used in science.

Measurement projects might be undertaken that involve the use of instructional time in both mathematics and science.

PRACTICAL ARTS

Measurement skills are highly used in many of the practical arts courses. Through cooperative planning, mathematics teachers can establish not only the skills required of students in these courses, but also relevant contexts in which the skills are applied. Integration with the practical arts should emphasize:

- metric units frequently used by students
- imperial units that the student may encounter in the workplace
- measurement tools used by the student
- technical drawings and patterns that the student must read and interpret.

COMMUNITY PARTNERSHIP OPPORTUNITIES

- Invite a home economist to speak to the class. Discussion may relate to:
  - home costs
  - comparative shopping
  - strategies for becoming a wise consumer.
- Plan a field trip to a local grocery store. Ask students to determine and compare unit prices on competing consumer products.
- Plan a field trip to a local utility company. Investigate:
  - strategies for monitoring utility consumption in the home
  - strategies for reading utility meters/gauges
  - utility rates and customer billing procedures
  - strategies for reducing utility consumption in the home.
- Visit a local hardware store/lumber yard/fabric store/hobby shop. Investigate the use of measurement in determining the quantity and price of various materials purchased at these stores.
- Ask students to interview their own care-givers regarding mathematical skills needed in the home, on the job, etc.

CALCULATING FOOD AND UTILITY COSTS

BASIC RESOURCE CORRELATION

Mathbase 1
- Chapter 2: Solving Problems
  - Guessing and Checking
- Chapter 4: Data Graphs
  - Interpreting Bar Graphs

SUGGESTED ACTIVITIES

1. Ask students (individually or in small groups) to prepare a list of food items they feel would be necessary to provide themselves with breakfast, lunch and dinner for a period of one week.

   Have students estimate their monthly food cost by:

   - pricing each food item on their list (through the use of store flyers or by visiting local supermarkets)
   - determining the total cost of all food items required for one week
   - extending the cost per week to an estimated cost per month.

   Encourage students to consult the Canada Food Guide to ensure that the meals they plan are healthful. Teachers may wish to plan this activity in cooperation with the teacher of the Health and Personal Life Skills class, and have students plan balanced meals that include foods from each of the food groups.

   Encourage students to compare their monthly food costs, and to make a list of ways in which money might be saved on food budgets.

2. Discuss the meaning of "unit price" and its importance in comparative shopping. Provide opportunities for students to demonstrate their understanding through different forms of expression:
   - through concrete/pictorial representation
   - through written description
   - by writing number sentences/equations
   - by designing a computer program for finding unit price.

   Teachers are encouraged to use a variety of strategies in developing the concept of "unit price" (see Using a Math Lab, "Ratio, Proportion and Percent" and Use of Technology, "The Calculator and Computer").

3. Discuss the application of mental arithmetic and estimation skills in consumer situations (e.g., comparative shopping, estimating total cost of items selected for purchase, verifying change received). Provide frequent opportunity for students to develop and practise these skills in simulated consumer situations (see Computational Facility and Estimation, "Developing Mental Arithmetic Skills/Strategies for Estimation").

4. Ask students to price the ingredients required for a recipe that they are given. The following procedure might be used:

   - By visiting a local supermarket (or using sale flyers), determine the unit price of each ingredient used in the recipe. (In the case of competing brand names, encourage students to select the brand that is least expensive.)

- Once back in the classroom, calculate the total cost of the recipe (e.g., multiply unit costs by the amount of each ingredient required and add the costs of individual ingredients).

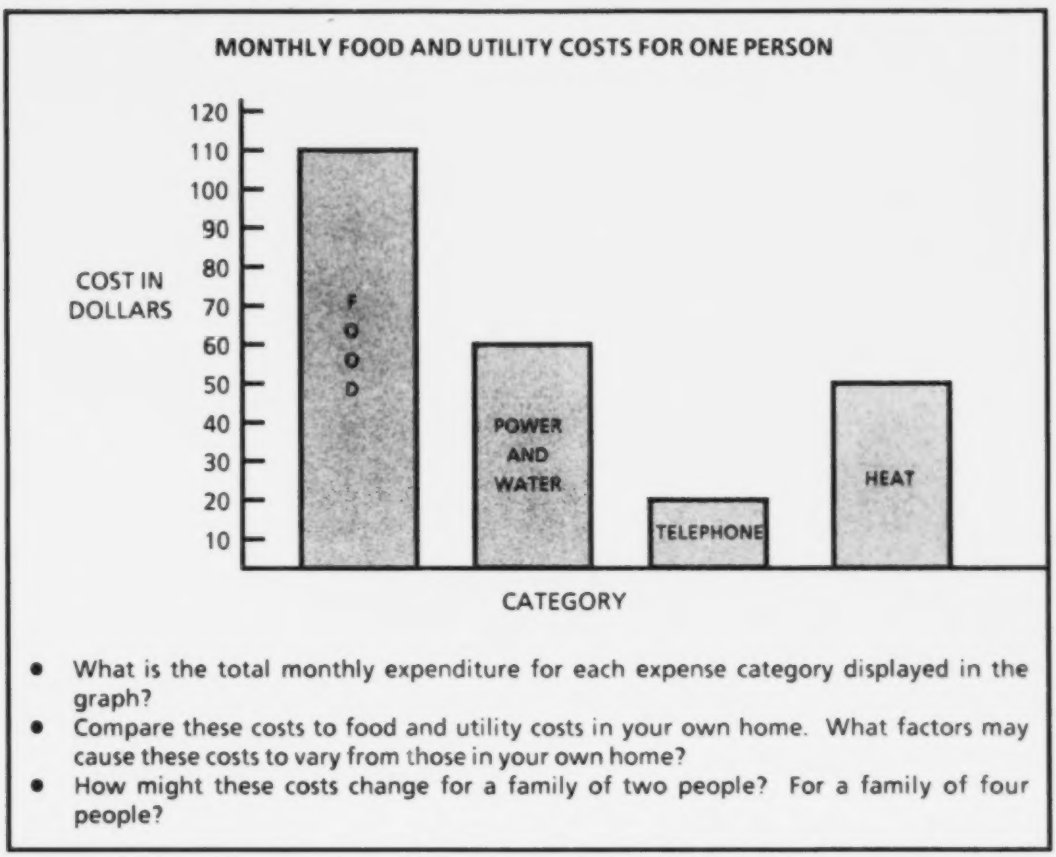
5. Compare the cost of dining out with eating at home by planning the following activity:

   - obtaining a menu from a local restaurant and asking students to determine the cost of a meal for one or more people
   - using supermarket flyers (or by visiting the local supermarket), estimate the cost of the same meal if prepared at home
   - summarizing the results of the investigation in table/chart form.

6. Ask students to interview their care-givers in order to determine regular home costs (e.g., monthly food costs and utility bills). Discuss factors that may cause monthly costs to vary:
   - number of family members
   - size/type of home
   - life style/personal habits
   - seasonal variations.

7. Discuss the use of bar/line/picture graphs in displaying/comparing various home costs. Ask questions that require students to interpret and analyze the information presented in these graphs.

CLARIFICATION/EXAMPLE



- What is the total monthly expenditure for each expense category displayed in the graph?
- Compare these costs to food and utility costs in your own home. What factors may cause these costs to vary from those in your own home?
- How might these costs change for a family of two people? For a family of four people?

8. Discuss the use of water/natural gas/electrical meters in monitoring the consumption of utilities in the home.

   Develop procedures for reading the scales/calibrations that are represented on utility meters (see Resource 1: Reading Scales).

   Encourage students to monitor utility consumption in their own homes by recording meter readings over a period of several days or weeks. Ask students to draw diagrams of each meter reading that is taken.

9. Plan a visit to a local utility company. (The visit might be cooperatively planned with the science class in conjunction with the theme "Using Energy and Machines".) Interview employees, and investigate:

   - various meters/gauges and units used in measuring utilities
   - utility rates and customer billing procedures
   - strategies for reducing utility consumption in the home.

USING MEASUREMENT SKILLS

BASIC RESOURCE CORRELATION

Mathbase 1
- Chapter 5: Decimals and Measurement
  - The Metre
  - The Centimetre
  - The Millimetre
  - Selecting the Best Unit
  - Perimeter

COMPUTER SOFTWARE

Fast Facts

SUGGESTED ACTIVITIES

1. Provide abundant opportunities for hands-on experience in estimating and measuring length, capacity and mass. A variety of strategies for developing skills of estimation and measurement are provided in the "Generic Strategies" section of this manual (see Using a Math Lab: "Geometry and Measurement").

   The guidelines which follow may be useful in developing appropriate thematic activities in estimation and measurement:

   - Use "mind pictures" or referents to facilitate estimation with units of length, capacity and mass.
   - Have students make their own metric tapes.
   - Ask students to estimate and measure:
     - the length of a room in metres
     - the width of a table in centimetres
     - the height of a window in centimetres
     - the capacity of familiar containers in mL and L
     - the mass of familiar objects in g and kg.

- Use the *Guinness Book of Records* and/or *Olympic Records* to make a bulletin board display answering questions of interest such as:
  - Who holds the high jump record?
  - What is the highest jump ever recorded?
- Assist students to develop a referent for the kilometre by using the following strategy:
  - measure off 100 metres
  - explain that if this distance is walked 10 times, the total distance travelled is one kilometre
  - relate this measure to the distance between two familiar locations in the community.
- Determine how many laps in the gym are equivalent to running one kilometre.
- Plan a walking rally or treasure hunt using given distances.
- Ask students to estimate and then measure/compare their own body measurements (e.g., chest size, arm length, height, foot size).
- Develop visualization skills that are useful in estimating measures through various guessing games (e.g., How many jelly beans in the jar?).
- Display household containers of various size and shape. Ask students to estimate and measure the capacity of these containers.
- Display various containers of familiar kitchen products. Ask students to estimate their mass and then measure mass as a check against estimates that have been made.
- Purchase a variety of produce, dry goods and boxed goods at the supermarket. Weigh the items purchased and discuss the following points:
  - How much does the packaging weigh?
  - What part of the total weight purchased is edible?
  - Are the weights indicated on package labels correct?

2. Develop the concept of perimeter through the use of geoboards/dot paper/grid paper (see Using a Math Lab, "Geometry and Measurement"). A blackline master for producing grid paper is provided as Resource 2: Centimetre Grid Paper.

   Ensure that students recognize perimeter as "distance around". Provide opportunity for students to:

   - determine the perimeter of familiar objects in the classroom
   - determine the perimeter of objects represented by diagrams/scale drawings
   - determine the quantity and cost of materials required for various home projects:
     - moulding for a room/window/door
     - trim for a sewing project
     - fencing material for the back yard.

3. Provide students with a scale drawing of the floor plan for a house/apartment (see Resource 3: A Sample Floor Plan). Ask questions about the scale drawing that will require students to measure and interpret the drawing:

   - Which room is the longest/shortest?
   - What is the entire length/width of the house?
   - What is the width of a doorway?
   - How much moulding would be required for the living room?
   - How much moulding is required for the windows in bedroom #1 if each window is one metre in height?

4. Ask students to draw a floor plan for their own house/apartment on the computer using LOGO in the draw mode (see Use of Technology, "The Computer"). Drawings should be based upon simple sketches and measurements that have been taken by students at home.

Help students to establish an appropriate scale for their drawings on the computer, and ask questions that will cause students to interpret the drawings they make:

- What is the entire length/width of your home?
- What is the perimeter of your bedroom?
- How many "turtle steps" must you walk to go from your bedroom to the kitchen?

5. Provide opportunities for students to follow a simple recipe involving measurement of mass and capacity. This activity can be planned cooperatively with a teacher in the practical arts, and might be performed in the food preparation laboratory.

   Assist students in following their recipes by discussing:

   - appropriate measuring devices
   - appropriate strategies for measuring food items by mass/capacity
   - the importance of "sequence" in the tasks performed.

AN EVENING'S ENTERTAINMENT

BASIC RESOURCE CORRELATION

Mathbase 1
- Chapter 10: Geometry: Shapes and Designs
  - Designs
  - Designs Using LOGO
  - Tiling Patterns

Mathbase 2
- Chapter 4: The Best of Entertainment

COMPUTER SOFTWARE

Math Strategies: Problem Solving

SUGGESTED ACTIVITIES

1. Provide opportunities for students to play a variety of games in which they practise and apply skills in mental arithmetic (e.g., dominoes, monopoly, cribbage). Encourage students to discuss and share strategies for mental arithmetic that are being used throughout each game (see Computational Facility and Estimation, "Developing Mental Arithmetic Skills").

   Additional activities that may be useful in developing competence in mental arithmetic are provided in Using a Math Lab, "Number Systems and Operations".

2. Read and interpret radio and television schedules. Have students complete charts and reports based on information obtained from these schedules. Each student might complete:

   - a summary of sports events/game shows telecast between 6:00 p.m. and 9:00 p.m. on weekdays only
   - a report on favourite television programs (including the day, time and channel for each program included).

   Ask students to prepare a personal schedule for watching television/listening to the radio for a period of one week.

3. Ask students to compare the cost of different forms of entertainment:

   - the cost of renting a video compared to going to the movie theatre
   - the cost of going to a hockey game compared to watching a game on television
   - the cost of ordering "fast foods" compared to snacks/meals prepared at home.

4. Develop and reinforce geometric concepts/skills by involving students in various hobbies/projects that require the use of geometric tools in creating patterns and designs. Students might:

   - create a wallpaper/fabric/floor covering design based on repetition of a line design or geometric pattern
   - design a logo for a favourite rock band using one- and two-dimensional figures/relationships
   - use a LOGO computer program to create a geometric design that is based on line relationships and geometric figures studied.

   Additional ideas for projects involving line design and geometric pattern are provided in Using a Math Lab, "Geometry and Measurement".

5. Develop skills in visual perception by providing puzzles that require students to recognize and apply geometric patterns and relationships. Students might be encouraged to work on these puzzles with family and friends in the evening, and to share strategies and outcomes with their peers the next day. Puzzles may involve experimentation with:

   - symmetry
   - polyminoes
   - tangrams
   - tessellations
   - mazes
   - paper folding.

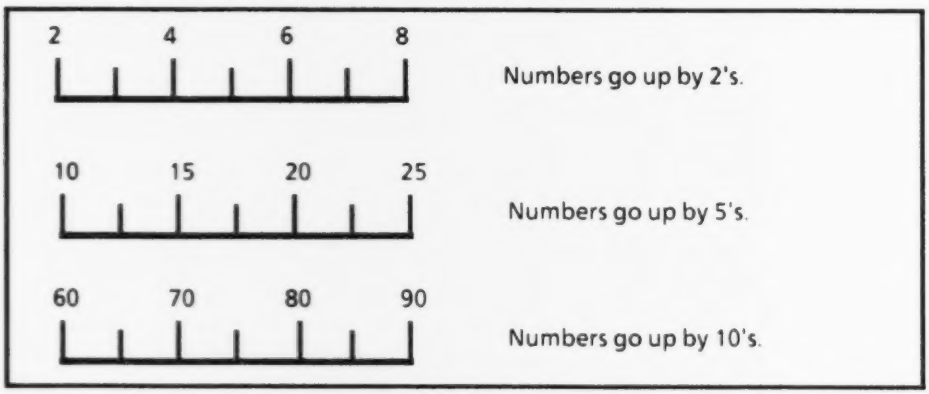
   A variety of books containing recreational puzzles of this nature can be obtained from local bookstores and libraries.

RESOURCE 1: READING SCALES

Before you read any scale, you must examine it carefully to see how it is calibrated.

- Look at the numbers written on the scale. They may not be sequenced in the usual order (e.g., 1, 2, 3, 4 and so on).

  Study the examples:

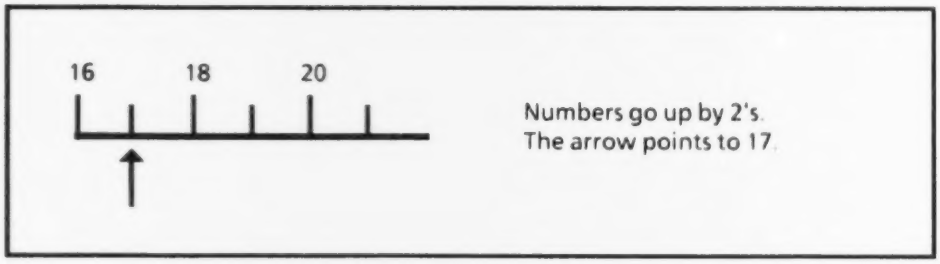


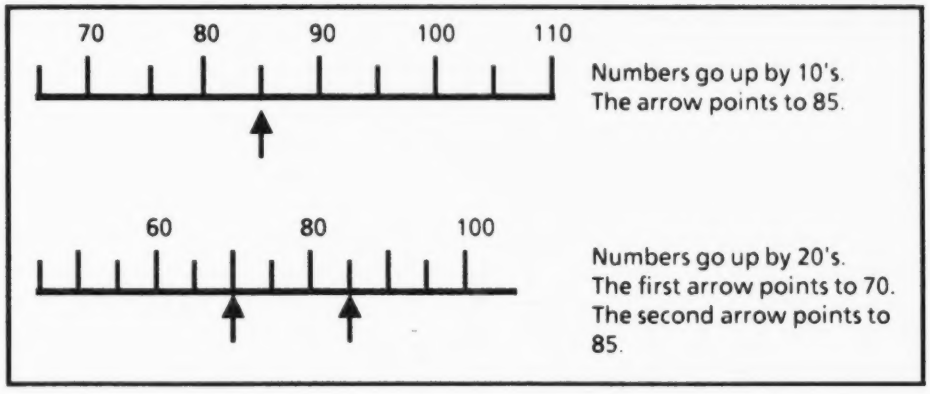
  Other scales may have numbers that go up by 20's, 50's, 100's, or 1000's.

- Look at the marks between the numbers. The numbers represented by these marks depend upon:

  - the number of spaces between two consecutive numbers
  - the difference between two consecutive numbers on the scale.

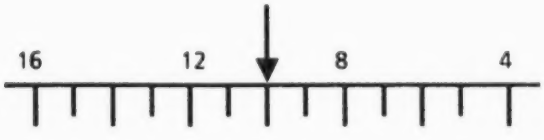
  Study the examples:





READING SCALES THAT GO BACKWARDS

Sometimes a scale appears to go backward. These scales must be read from right to left, as in the example below.

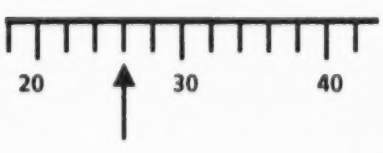


"Backward" scales are often found on meters that are turned by gears (e.g., a water meter or an electric meter in a house).

READING MORE DIFFICULT SCALES

An easy way to figure out the value of each space on a scale is to divide the difference between two numbers on the scale by the number of spaces between them.

Example 1:



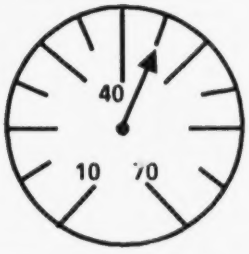
The difference between 20 and 30 is 10.

There are 5 spaces between 20 and 30.

Therefore, each space represents $\frac{10}{5}$ or 2 units.

Each space represents 2. The arrow indicates 26.

Example 2:



The difference between 40 and 70 is 30.

There are 6 spaces between 40 and 70.

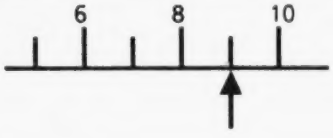
Therefore, each space represents $\frac{30}{6}$ or 5 units.

Each space represents 5. The arrow indicates 45.

EXERCISES

1. Look carefully at each scale. Then answer the questions.

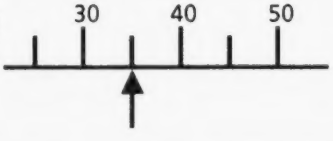
(a)



a. The numbers written on the scale go up by _____.

b. The arrow points to the mark that means _____.

(b)



a. The numbers written on the scale go up by _____.

b. The arrow points to the mark that means _____.

(c)

a. The numbers written on the scale go up by _____.

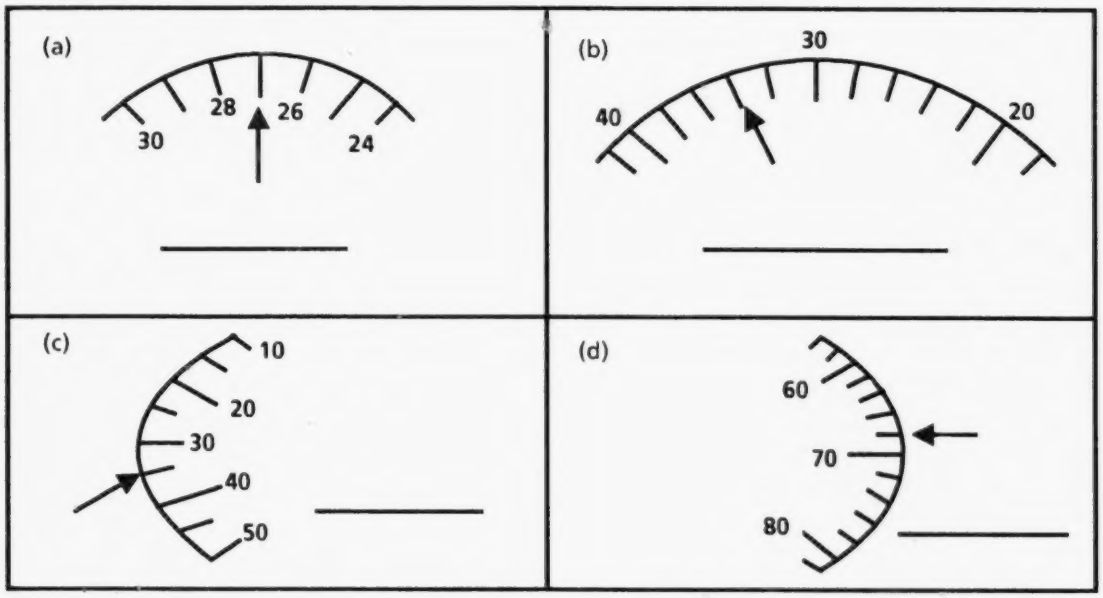
b. The arrow points to the mark that means _____.

(d)

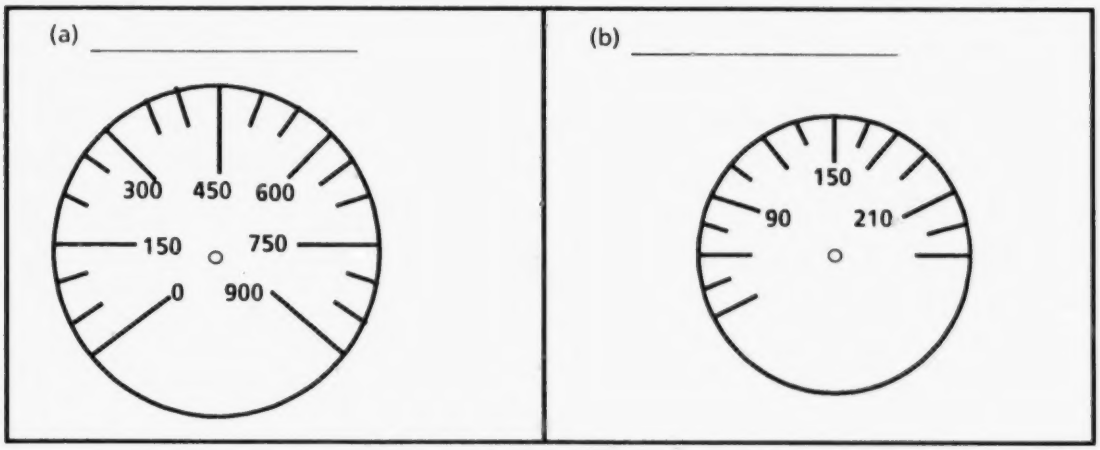
a. The numbers written on the scale go up by _____.

b. The arrow points to the mark that means _____.

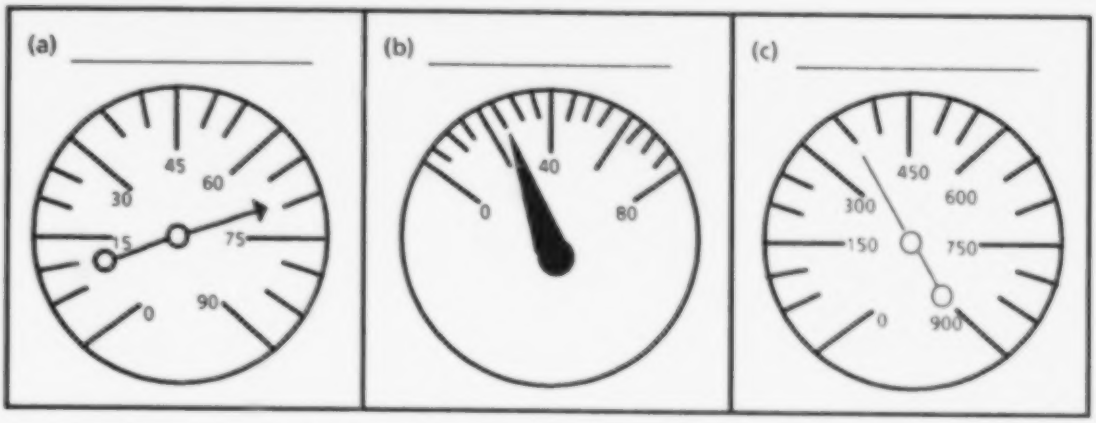
2. Read each of the following scales.



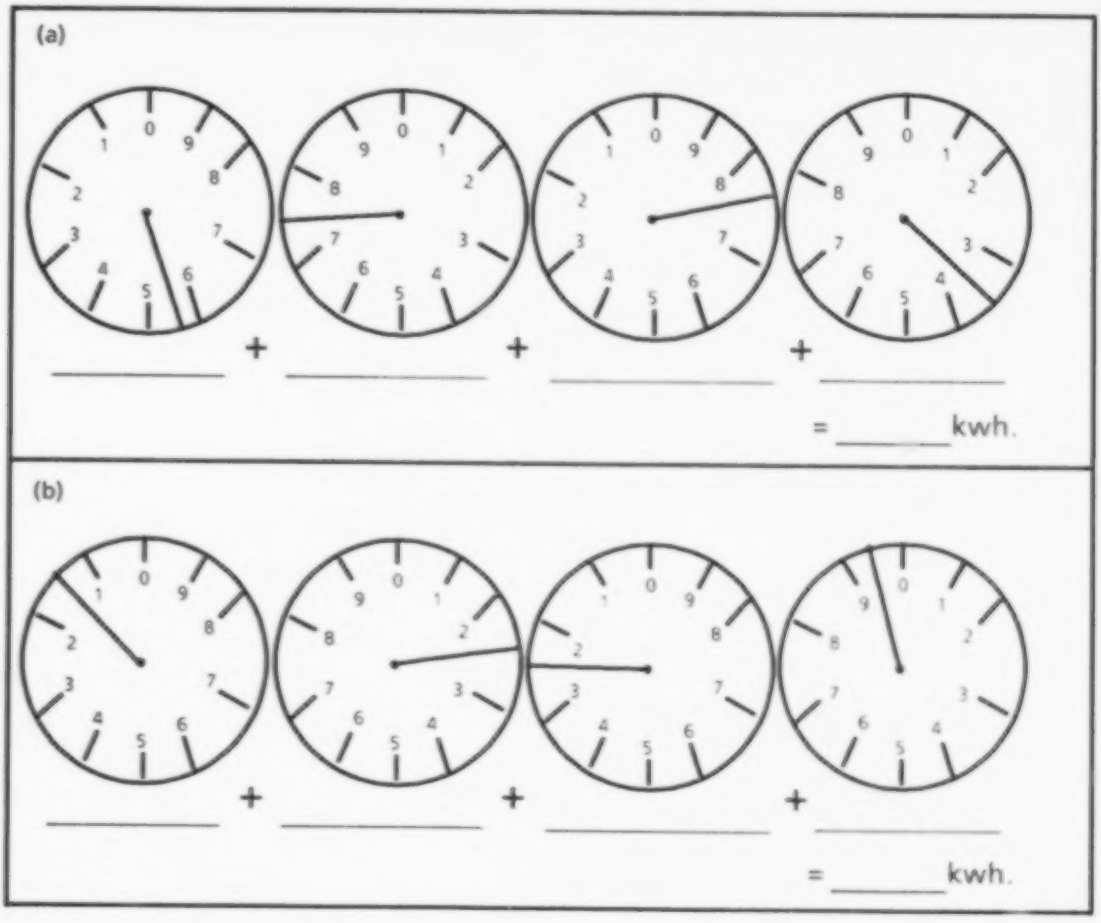
3. Give the value of each space on the scales illustrated below.



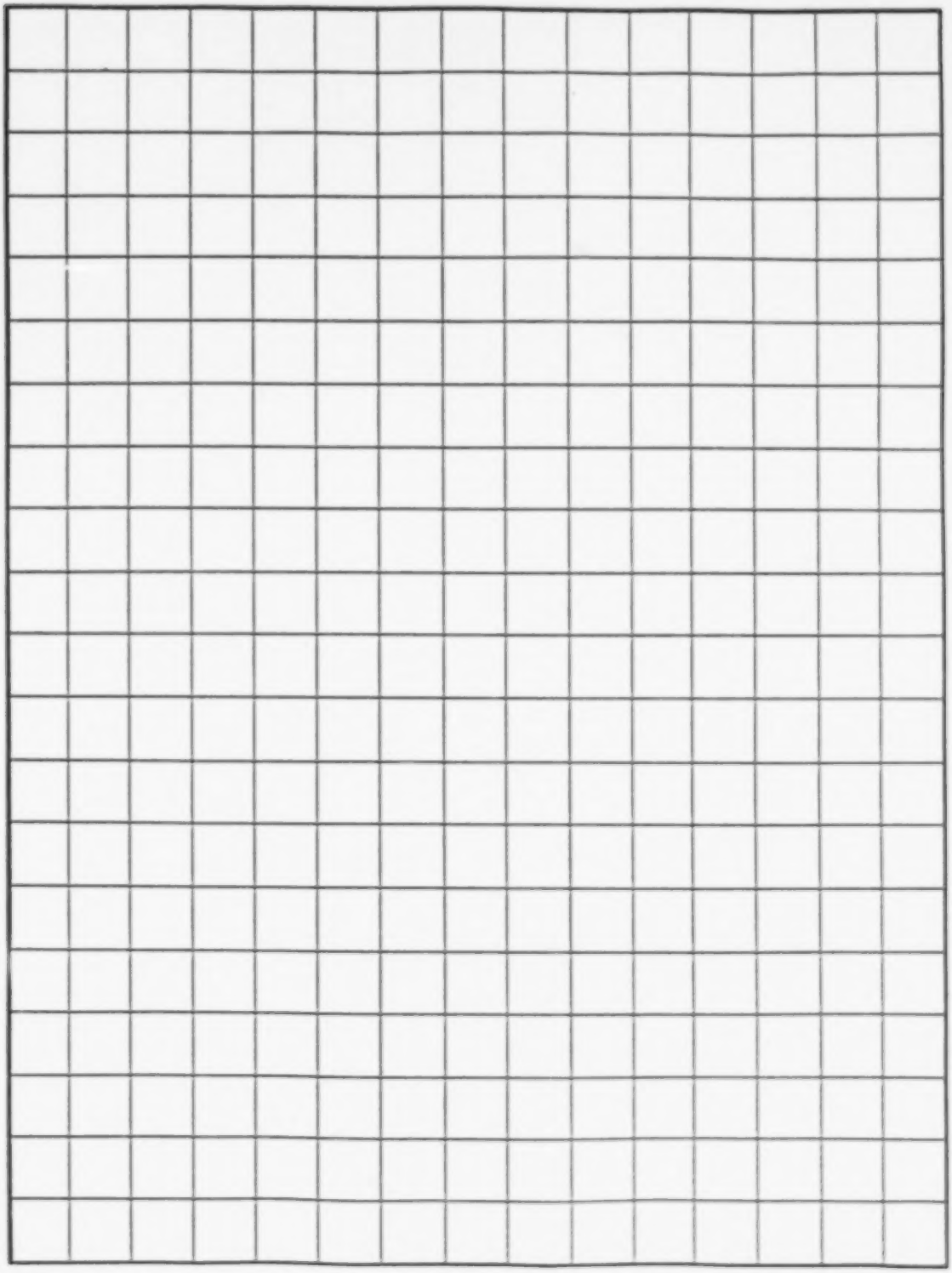
4. Give the reading indicated on each of the following scales.



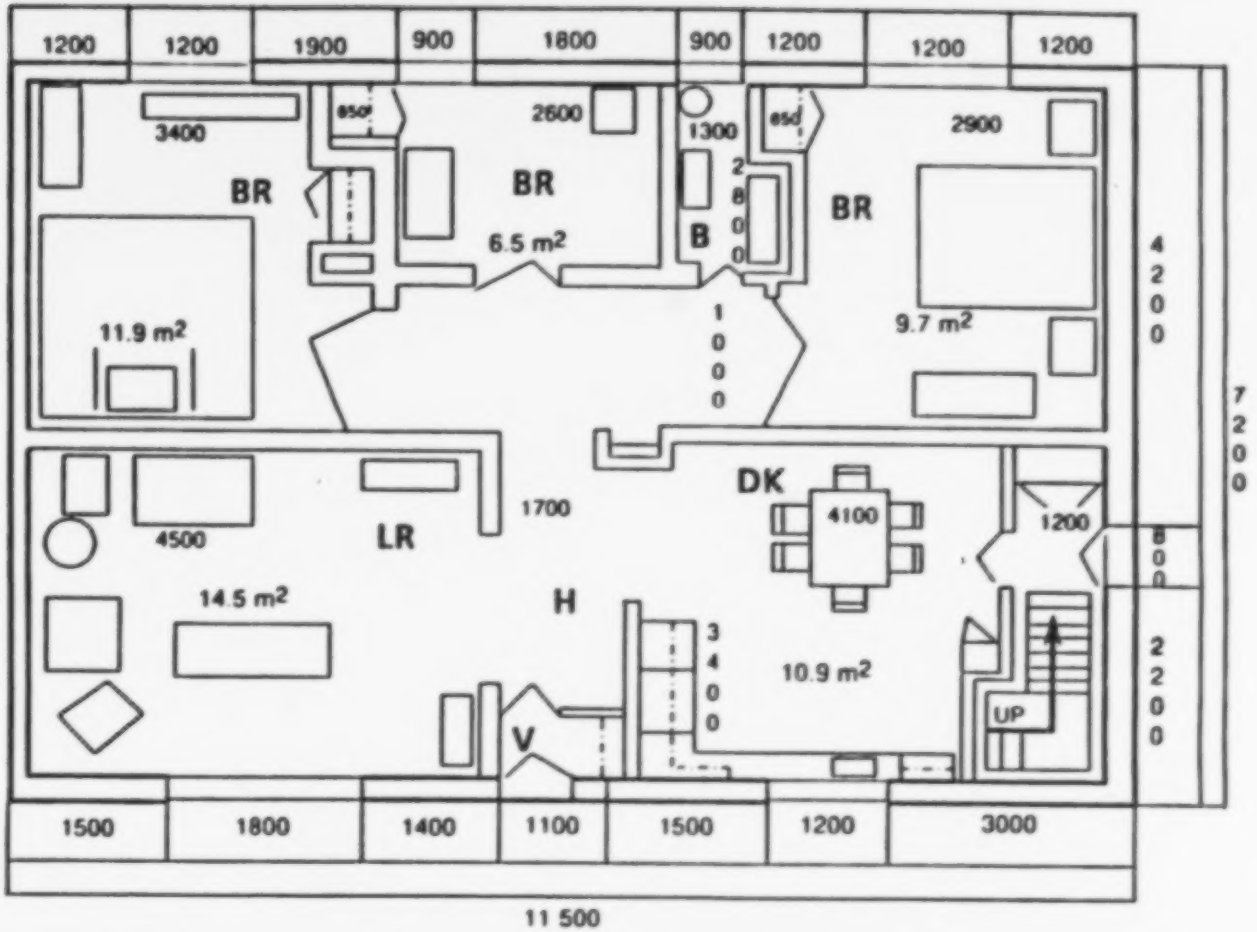
5. Give the readings indicated on each of the following electric meters.



RESOURCE 2: CENTIMETRE GRID PAPER



RESOURCE 3: A SAMPLE FLOOR PLAN



NOTE: All dimensions are given in millimetres unless otherwise indicated

I.O.P. MATHEMATICS 8

TRAVEL & RECREATION

SUB-THEMES

- USING MAPS AND TRAVEL SCHEDULES
- COMPARING RECREATIONAL COSTS
- STATISTICS IN SPORTS

RATIONALE

"Travel and Recreation" is a theme that most students should be particularly interested in as it involves their use of leisure time. Opportunity will be provided for students to investigate various aspects of the activities that frequently occupy their leisure time after school and during holidays.

Travel schedules and maps are a part of everyone's life, and become essential whether planning a trip to a friend's home, to the ski slope or to Hawaii. Recreational activities affect our health and lifestyle, and should be selected after considering various factors related to personal circumstances. Many students spend some of their leisure time in sports, either as active participants or as observers, and will benefit from a study of statistical terms frequently used in familiar sporting events.

Teachers are encouraged to modify thematic learning objectives in meeting the needs of individual students and according to circumstances within the local community. Many opportunities exist throughout the theme for integrating mathematical instruction with activities undertaken in other subject areas (e.g., social studies, language arts). The local community contains a variety of resources that will provide students with real life experiences related to many of the learning objectives.

Generic strategies that are particularly relevant to the learning objectives and activities outlined in this theme can be found in:

- Problem Solving
- Computational Facility and Estimation
- Using a Math Lab.

THEMATIC OBJECTIVES

USING MAPS AND TRAVEL SCHEDULES

- Identifies symbols on a road map.
- Locates streets and towns on a road map, using map coordinates.
- Determines the distance between two locations on a road map.
- Estimates the time required to travel a given distance represented on a road map.
- Gives directions for a trip route using a road map.
- Reads/interprets bus, train and airline schedules.
- Estimates the cost of a short trip, considering transportation, meals, accommodation and incidental expenses.

COMPARING RECREATIONAL COSTS

- Maintains a personal record of expenditures made for recreational activities.
- Compares equipment and participation costs for different sports/recreational activities (e.g., hockey, tennis, swimming, jogging, skiing).
- Monitors personal time spent in various recreational/leisure activities.
- Develops a strategy for selecting appropriate recreational/leisure time activities.

STATISTICS IN SPORTS

- Reads/interprets numeric sports information communicated by the media:
  - schedules
  - rankings
  - game results
  - player statistics.
- Calculates statistics related to specific sports:
  - rushing, passing and receiving averages
  - goals against, points per game and plus-minus ratings
  - batting averages, earned-run averages
  - time differences, rate of travel
  - measures of hitting/throwing/shooting success.
- Collects, records and displays sports information in table or chart form.

CONTEXT FOR INSTRUCTION

PROBLEM SOLVING

- Uses a problem-solving strategy in planning/budgeting for travel.
- Investigates/compares equipment and participation costs for different sports/recreational activities.
- Uses a problem-solving strategy in selecting recreational/leisure time activities that are appropriate to personal circumstances. Considers factors such as cost, availability of facilities and benefits to health in selecting activities.
- Solves routine problems involving sports statistics.

USE OF TECHNOLOGY

- Uses a calculator iteratively.
- Uses travel schedules and sports schedules.
- Uses the media to collect information related to travel, recreation and sports.
- Recognizes the use of the computer in compiling statistics and schedules.

COMPUTATIONAL FACILITY AND ESTIMATION

- Develops/reinforces basic computational algorithms.
- Performs computations with a calculator on a regular basis.
- Uses mental arithmetic when comparing sports statistics.
- Rounds as appropriate during calculations.
- Uses estimation in comparing the cost of different travel methods/recreational activities.
- Estimates travel distances and travel times represented on a map.
- Uses estimation in checking the reasonableness of computational results.

SUPPORTING CONCEPTS, SKILLS AND ATTITUDES

Number Systems and Operations	Ratio, Proportion and Percent	Geometry and Measurement	Data Interpretation and Display	Algebra

NUMBER SYSTEMS AND OPERATIONS

- Reads/writes/orders decimals to the thousandth's place.
- Counts by multiples of 2, 3, 4, 5, 6, and 12 in making tables/charts.
- Performs basic operations with whole numbers and decimals.
- Applies rules for the order of operations.
- Compares/orders fractions in applications.
- Identifies and determines equivalent fractions.
- Recognizes and expresses fractions in simplest form.

RATIO, PROPORTION AND PERCENT

- Recognizes ratio as ordered pairs of numbers showing comparison.
- Generates equivalent ratios using single-digit whole number constants.
- Recognizes proportions as statements about equivalent ratios.
- Determines the missing component in a proportion using the common factor/multiple method.
- Demonstrates the concept of percent as a ratio indicating parts out of 100.
- Uses ratio skills in interpreting "scale" on a road map.

GEOMETRY AND MEASUREMENT

- Identifies and distinguishes between horizontal, vertical, parallel, perpendicular, and intersecting lines.
- Estimates/calculates distance in kilometres between locations on a road map.
- Estimates/measures/records time on a 12-hour and 24-hour clock.
- Uses National Standards for numeric dating.

DATA INTERPRETATION AND DISPLAY

- Reads/interprets information displayed in list, table and chart form.
- Collects and records sports data using frequency tables/tally sheets.
- Uses charts/tables to display data.
- Reads and interprets data displayed in bar/picture/line graphs.

ALGEBRA

- Distinguishes between the use of variables and constants in concrete situations.
- Interprets/uses equations that describe practical situations (e.g., $d = rt$).
- Uses formulas to calculate sports statistics.

INTEGRATION ACTIVITIES

LANGUAGE ARTS

- Gives oral and written directions (i.e., describes a travel route from point A to point B).
- Uses research skills to gather data related to travel, recreation and sports.
- Uses reporting and note-taking skills to summarize data that has been gathered.
- Investigates the use of leisure time in the theme "How Are You Today?".

SOCIAL STUDIES

- Uses city maps and Alberta road maps.
- Estimates/calculates distances on a road map.
- Investigates local recreational facilities in the theme "You and Your Physical/Cultural Community".

SCIENCE

- Examines the effect of various recreational/leisure time activities on the environment.

PRACTICAL ARTS

- Identifies potential travel destinations and recreational facilities in the theme "Smile, You're a Tourist Attraction!".

COMMUNITY PARTNERSHIP OPPORTUNITIES

- Visit/write a local tourist bureau or travel agency and request travel information (e.g., city maps, Alberta road maps, brochures describing potential travel destinations or local recreational facilities).
- Visit a local bus/train/airline terminal and request travel schedules. Observe the use of 24-hour time notation in monitoring departure and arrival times.
- Invite representatives from local recreational/fitness clubs to discuss:
  - facilities/equipment available for use
  - user costs.
- Investigate local recreational opportunities that are of little or no cost to the participant.
- Invite a guest speaker from the local newspaper/television station/radio station to discuss the use of statistics in reporting the results of sports events.
- Attend a local sports event. Collect and record data about the event. Summarize game results through the use of appropriate statistics.

USING MAPS AND TRAVEL SCHEDULES

BASIC RESOURCE CORRELATION

Mathbase 1

- Chapter 1: Understanding Numbers
  - Numbering Within a Building
  - Coding: The Canadian Postal Code
  - Using Numbers on the Road
- Chapter 5: Decimals and Measurement
  - Place Value
  - The Kilometre
- Chapter 12: Geometry Around Us
  - Parallel Lines
  - Perpendicular Lines

SUGGESTED ACTIVITIES

1. Provide students with a map of the local area. Discuss the meaning of symbols used on the map. Ask students to identify map coordinates for:

   - their homes
   - the homes of other members of the class
   - the school
   - major shopping centres
   - recreational facilities.

   Extend map-reading activities by asking students to describe travel routes from their homes to other locations on the map. Directions might be given orally or through written expression. (Teachers may have students work in pairs. One student can give the directions while the second student follows these directions on the map.)

2. Play the game of "20 Questions".

   Game Rules:
   Students play the game in pairs. One student picks a "hiding spot" somewhere on a map. The other student must find this hiding spot by asking as few questions as possible. Questions that are asked must be answered only by "yes/no" or "north/south/east/west". When the hiding spot has been found, the players reverse their roles. The winner is the student who is able to find the hiding spot by asking the fewest questions.

3. Provide students with a road map of Alberta and a distance chart. Ask students to plan a road trip between two points in the province. Travel plans should include:
   - the route to be taken
   - major centres that will be visited while travelling
   - distances between major centres
   - total distance of the trip
   - approximate travel time for the trip, given an average travel rate of 60 km/h.

4. Discuss the use of "referents" in estimating distances on a road map (see Using a Math Lab, "Geometry and Measurement").

   Example:
   Assume that the distance between Calgary and Edmonton is 300 kilometres. Using this referent and an Alberta road map, ask students to:

   - estimate the distances between other points on the road map
   - estimate the time required to travel between these points, given a travel rate of 60 km/h.

5. Examine travel schedules and fares obtained from local bus, train and airline companies. Assist students to read/interpret the symbols. Discuss the use of 24-hour time notation.

   Use travel schedules and fares to compare alternative methods/costs of travel for a simulated trip (e.g., from Lethbridge to Edmonton) on a given day at a given time.

6. Visit a local travel agency or invite a local travel agent into the classroom. Ask the travel agent to discuss:

   - appropriate procedures for reading travel schedules
   - the use of 24-hour time notation
   - the use of computers in the travel industry.

7. Provide opportunity for students to plan a short trip. Through the use of travel information they gather, students should determine:

   - a destination
   - a method of travel
   - a travel route
   - dates/times for departure and return
   - approximate travel costs, including transportation, meals, accommodation and incidental expenses.

   Additional ideas for a project of this nature are provided in Using a Math Lab, "Project Work".

COMPARING RECREATIONAL COSTS

BASIC RESOURCE CORRELATION

Mathbase 1

- Chapter 4: Data Graphs
  - Displaying Data: Line Graphs
- Chapter 5: Decimals and Measurement
  - Rounding
  - Adding Decimal Numbers
  - Subtracting Decimal Numbers
  - Multiplying and Dividing Decimal Numbers by Whole Numbers
  - Using Your Skills

COMPUTER SOFTWARE

Fast Facts

SUGGESTED ACTIVITIES

1. Through discussion, identify potential recreational/leisure time activities for students in the class. By asking appropriate questions, encourage students to consider the following factors as they relate to the activities identified:

   - cost of equipment (e.g., running shoes, racquets, appropriate clothing)
   - cost of facilities where appropriate (e.g., club fees, user fees)
   - personal time commitments that are required
   - availability of suitable facilities in the local community
   - effect of the activity on personal health and lifestyle.

   Summarize the results of discussion on a large wall chart, where the most common leisure time activities are placed in rank order according to the:

   - cost to the participant
   - potential benefit to personal health and lifestyle.

2. Ask students to monitor their recreational activities on a daily basis over a given period of time (e.g., one or two weeks). Provide each student with a profile on which to record the:

   - types of activities they are involved in each day
   - money spent on each activity
   - time spent on each activity
   - fraction/percent of leisure time spent on each activity.

   Results might be summarized after one or two weeks by asking students to determine the:

   - total cost of all recreational activities
   - fraction/percent of leisure time spent on these activities.

Encourage students to share their results, and to identify those recreational routines that likely have the most positive impact on personal health and lifestyle for money spent. File individual profiles for future reference in this theme.

3. Ask each student to identify a sport/recreational activity in which they would like to participate at some future time in their life. Have each student conduct research on this sport/activity, to determine the:

   - cost of necessary equipment and facilities
   - potential effects of the sport/activity on health and lifestyle.

4. Invite a physical education teacher/sports coordinator into the classroom to discuss:

   - costs related to participation in various school sports (e.g., equipment costs, league fees)
   - potential health benefits/health risks that are related to various school sports.

5. Visit a local recreational facility/fitness club. Ask the manager to explain:

   - facilities/equipment/services
   - membership costs/user fees
   - contracts/agreements between facility and user.

   Following the visit, debate the advantages/disadvantages of this kind of facility when compared to other community facilities that are of little or no cost to the user (e.g., local track club, community swimming pool).

6. Ask students to re-examine their profiles of recreational activities completed earlier (see Activity #2). Encourage students to evaluate their use of leisure time and possibly modify their recreational routines in ways that may be more appropriate to personal health and circumstance.

   Assist students in this task by modelling appropriate problem-solving and decision-making strategies (see Problem Solving, "Using Strategies to Solve Problems").

STATISTICS IN SPORTS

BASIC RESOURCE CORRELATION

Mathbase 2
- Chapter 8: Sports Math

SUGGESTED ACTIVITIES

1. Discuss the use of the term "average" in sports:

   - a goalie has a "goals against" average
   - a pitcher has an "earned run" average
   - a batter has a "batting" average.

   Review the algorithm for finding average. Ask students to read sports articles in the newspaper and underline situations in which an average has been reported.

2. Develop and reinforce basic concepts/skills in ratio and percent by using strategies suggested in Using a Math Lab, "Ratio, Proportion and Percent". Student ability to interpret statistics in sports will depend upon their comprehension of numbers, operations, ratio and percent.

3. Ask each student to examine the sports section of the local newspaper. Identify and discuss instances in which statistics are used to report the:

   - results of a game or event
   - standing of a team
   - statistics of a player.

   Working individually or in small groups, have students select a game summary provided in the newspaper. Using this summary, make an attractive display by:

   - placing the summary on a piece of poster paper
   - listing/explaining each of the statistics used
   - formulating five questions about the game described in the news article.

   Ask students to exchange their displays and answer the questions that have been asked.

   Sample displays have been provided in Resource 1: The Great Gretzky Returns, and in Resource 2: WHL Standings.

4. Have each student (or group of students) identify a sports hero. Prepare a bulletin board display of each hero, illustrating:

   - his or her accomplishments
   - statistical data related to the accomplishments.

5. Invite the coach of a local team to discuss the use of statistics in coaching. Ask the speaker to explain:

   - methods of gathering information about players/teams
   - the kinds of statistics they feel are important
   - inferences that can be made from these statistics.

6. Attend a local sports event or watch a game on television. Ask students to collect and record data pertinent to the event (e.g., number of shots on goal, attempted baskets and number made) using tally sheets and frequency tables. Once data has been collected, it should be summarized using appropriate statistics (e.g., ratio, percent, average). Display the results of the event in a table or chart.

RESOURCE 1: THE GREAT GRETZKY RETURNS

A summary of the Oilers/Kings game on October 19, 1988 is provided below. At this game, Wayne Gretzky made his first appearance in Edmonton wearing a Los Angeles uniform.

Answer the following questions using the game summary.

1. At what time was the first goal scored?

2. How may minutes/seconds later was the second goal scored?

3. Who scored the most:
   a. goals?
   b. assists?
   c. total points?

4. Calculate Fuhr's "goals against" average for each of the three periods and for the game.

OILERS 8 – KINGS 6[1]

FIRST PERIOD

1. Edmonton, Anderson 1 (Carson, Simpson) 3:25
2. Los Angeles, McSorley 4 (Allison, Fenton) 4:56
3. Edmonton, Simpson 4 (S. Smith, Anderson) 13:47
4. Edmonton, Lacombe 5 (Huddy) 18:10

PENALTIES - Huddy, Edm, 0:50; Messier, Edm, 6:14; Duchesne, LA, 8:59; Nicholls, LA, Tikkanen, Edm, 10:54; Duchesne, LA, 15:33

SECOND PERIOD

5. Edmonton, Messier 2 3:40 (sh)
6. Los Angeles, Duchesne 2 (Nicholls, Gretzky) 6:13 (sh)
7. Edmonton, Joseph 1 (MacTavish) 12:21
8. Los Angeles, Nicholls 5 (Robitaille, Walters) 12:46
9. Edmonton, Messier 3 (S. Smith, Kurri) 14:01
10. Los Angeles, Nicholls 6 (Taylor, Gretzky) 16:22 (pp)

PENALTIES - Carpenter, LA, 0:36; Hammond, Edm, 3:09;Tookey, LA, 4:52; Muni, Edm, 9:39; Carson, Edm, 14:35; Baumgartner, LA, 18:30

THIRD PERIOD

11. Edmonton, Adams, 2 (MacTavish, Lacombe) 1:27
12. Los Angeles, Fenton 1 (Tonelli, Allison) 2:11
13. Los Angeles, Robitaille 7 (Nicholls, Duchesne) 9:37
14. Edmonton, Larson 1 (Kurri, MacTavish) 15:11 (sh)

PENALTIES - Joseph, Edm, 4:57; Anderson, Edm, (high-sticking, major, game misconduct) 12:15; Larson, Edm 19:53

SHOTS ON GOALS BY:

Los Angeles - 11-13-13 - 37
Edmonton - 6-9-6 - 21

Goal - Los Angeles, Healy (L, 3-2-0); Edmonton, Fuhr (W, 1-1-2)
Powerplay conversions - Los Angeles, 1-9; Edmonton, 0-5
Referee - Koharski **Attendance** - 17,503.

5. What is Healy's win-loss record?

   Express this record as a ratio.

   What percent of his games played has Healy won?

6. How much time was left in the game when Anderson received his game misconduct?

[1] Reprinted by permission of *The Edmonton Sun*.

RESOURCE 2: WHL STANDINGS

Standings in the Western Hockey League are provided below. Use these standings to answer the questions.

1. Calculate the win/loss ratio for each team.

2. Calculate the "goals for" and "goals against" average for each team.

3. Is a team's standing better represented by a "win/loss" ratio or by a "goals for/against" average? Why?

4. How many more goals has Saskatoon scored than Kamloops?

WHL Standings[1]

EAST DIVISION

	G	W	L	T	F	A	P
S. Current	8	8	0	0	55	34	16
Saskatoon	8	5	3	0	43	33	10
P. Albert	7	4	2	1	31	26	9
Lethbridge	8	4	4	0	41	39	8
Regina	9	3	5	1	40	44	7
Med. Hat	8	3	5	0	44	45	6
Brandon	7	3	4	0	25	29	6
Moose Jaw	8	1	7	0	25	53	2

WEST DIVISION

Portland	6	5	1	0	33	28	10
Seattle	6	4	2	0	27	24	8
Victoria	7	4	3	0	38	27	8
Kamloops	7	3	4	0	34	29	6
Tri-Cities	8	3	5	0	24	28	6
Spokane	7	1	6	0	26	47	2

YESTERDAY

Moose Jaw at Regina
Saskatoon at Lethbridge
Seattle at Kamloops
Spokane at Victoria
Tri-Cities at Prince Albert

TUESDAY

Saskatoon 6, Tri-Cities 2

5. Using the information provided in the chart, predict what the number of wins and losses for Lethbridge would be over an 80 game season?

6. What is the average number of goals scored per game by the Tri-Cities team?

[1] Reprinted by permission of *The Edmonton Sun*.

I.O.P. MATHEMATICS 9
MANAGING YOUR MONEY

SUB-THEMES

- BANKING
- GETTING VALUE FOR MONEY SPENT
- BUDGETING

RATIONALE

Many students in junior high school are receiving some type of income, and need to develop strategies for managing the money they receive. This theme will enable students to:

- extend their understanding of banking procedures
- develop mathematical skills required in comparative shopping and spending money wisely
- plan for the saving and spending of allowances and money earned

Abundant opportunities exist for students to review basic computation and estimation skills. Thematic activities will permit teachers to diagnose individual student strengths and weaknesses, and to provide direct or focussed instruction as required in remediating student difficulties.

Cooperative planning with other teachers will provide ways of coordinating thematic activities in mathematics with topics studied in other subject areas. Such conferencing will ensure that instruction provided in mathematics will reinforce and enhance learning taking place within other disciplines. Many opportunities exist for experiential learning through community partnerships, classroom role playing and the use of manipulative materials.

Teachers are encouraged to reference the "Generic Strategies" section of this manual when planning for instruction. Strategies particularly relevant to the learning objectives and activities outlined in this theme can be found in:

- Using a Math Lab
- Computational Facility and Estimation
- Problem Solving
- Use of Technology

THEMATIC OBJECTIVES

BANKING

- Recognizes major types of chequing and savings accounts:
  - personal chequing account
  - regular (non-chequing) savings account
  - daily interest savings account
  - daily interest chequing/savings account.
- Understands and uses routine banking forms:
  - completes deposit/withdrawal slips
  - writes cheques
  - interprets bank statements.
- Maintains/verifies a current balance in a bank passbook.
- Recognizes how interest rates for savings differ from interest rates for loans.
- Recognizes and compares current interest rates for savings and for loans.
- Calculates yearly interest on money saved/money borrowed when given the principal and rate of interest (e.g., $I = p \times r$).
- Determines interest for periods other than one year using an interest table.

GETTING VALUE FOR MONEY SPENT

- Counts sums of money/makes correct change for amounts up to $100.
- Estimates/calculates total cost of a number of items purchased and verifies change received.
- Develops strategies for comparative shopping and determining the "best buy":
  - determines/compares unit prices
  - determines/compares discounts and sale prices (when given discount rate as a fraction or percent)
  - considers the effect of "coupon discounts" on the cost of consumer items.
- Recognizes the advantages and disadvantages of making credit/installment purchases.

BUDGETING

- Maintains a record of how personal income is spent.
- Compares the portion of personal income spent on "necessities" with the portion spent on "pleasure".
- Illustrates personal spending habits in picture/bar/line graph form.
- Maintains a balance sheet based upon personal income and expenditure.
- Recognizes the value of a regular saving plan to provide for future needs.
- Plans a budget based upon present income and anticipated expenditures that will accommodate some future purchase.

CONTEXT FOR INSTRUCTION

PROBLEM SOLVING

- Selects a type of bank account most appropriate to personal goals and needs.
- Determines/compares the effect of simple interest on money saved and money borrowed over varying periods of time.
- Uses a problem-solving strategy to determine the "best buy" (e.g., considers unit price, discounts and sale price, coupon discounts).
- Identifies a personal goal and plans a budget that will assist in realizing this goal.

USE OF TECHNOLOGY

- Uses the calculator iteratively.
- Determines interest through use of the calculator and/or computer.
- Uses interest tables.
- Recognizes applications of the computer in banking.

COMPUTATIONAL FACILITY AND ESTIMATION

- Uses basic computational algorithms.
- Uses the calculator on a regular basis.
- Uses mental arithmetic in consumer situations.
- Rounds to the nearest dollar/cent.
- Estimates sums and differences in consumer contexts.
- Estimates discount and sale price.
- Uses estimation to check the reasonableness of calculations and computation.

SUPPORTING CONCEPTS, SKILLS AND ATTITUDES

Number Systems and Operations	**Ratio, Proportion and Percent**	Geometry and Measurement	**Data Interpretation and Display**	**Algebra**

NUMBER SYSTEMS AND OPERATIONS

- Reads, writes, and orders whole numbers and decimals.
- Identifies place value from .001 to 100 000 in context.
- Rounds whole numbers and decimals as required.
- Performs basic operations with whole numbers and decimals.
- Performs a sequence of operations using correct order.
- Uses a fraction to represent part of a whole.
- Converts fractions into decimals.
- Recalls decimal equivalents for commonly used fractions.
- Applies integer skills as required in budgeting and in interpreting bank statements.

RATIO, PROPORTION AND PERCENT

- Recognizes unit price as a ratio.
- Uses ratio to calculate unit/multiple price.
- Expresses whole number percents as ratios and decimals.
- Calculates and estimates percent of a number in applications (e.g., discount, interest).
- Recalls fraction, decimal and percent equivalents for halves, quarters and tenths.
- Determines other fraction, decimal and percent equivalents through use of the calculator.

GEOMETRY AND MEASUREMENT

- Recognizes common metric units of length, mass and capacity.
- Converts measurements among commonly used metric units of length, mass and capacity.
- Recognizes the relationship between days/weeks/months/year.
- Uses National Standards for numeric dating.

DATA INTERPRETATION AND DISPLAY

- Reads/interprets information presented in table/chart form.
- Uses tables/charts to record data.
- Displays data in the form of picture/bar/line graphs.

ALGEBRA

- Uses variables/mathematical expressions to describe practical situations.
- Uses the formula $I = p \times r$ to solve yearly simple interest problems.
- Develops/applies formulas for finding unit price/multiple price/sale price.

INTEGRATION ACTIVITIES

LANGUAGE ARTS

- Discusses personal goals. Uses note-taking and reporting skills to formulate plans upon which to base a budget.
- Reads and interprets information gathered from financial institutions on different types of bank accounts.
- Develops vocabulary related to budgeting and banking.
- Reads and writes numbers to 100 000.
- Reads, interprets and completes a variety of banking forms:
  - deposit/withdrawal forms
  - cheques
  - bank statements.

SOCIAL STUDIES

- Recognizes the advantages/disadvantages of the use of various financial institutions and types of bank accounts.
- Recognizes the advantages/disadvantages of using credit.
- Recognizes the similarities/differences between a "personal" budget and a "government" budget.

PRACTICAL ARTS

- Determines unit price/multiple price in planning a project and ordering supplies.
- Determines percent discount and mark-up in establishing charges for service rendered and products produced.
- Counts money and makes change in situations where the amount tendered is less than $100.
- Applies budgeting skills in selecting and costing materials used in a variety of projects.

COMMUNITY PARTNERSHIP OPPORTUNITIES

- Invite guest speakers from local banks, credit unions, and loan companies to discuss:
  - services offered by these institutions
  - features of different types of accounts
  - current interest rates
  - use of banking forms.
- Visit a local financial institution and:
  - interview personnel regarding services offered
  - observe the use of banking forms
  - examine the use of computers and other technologies.
- Visit a variety of local businesses involved in the retail sale of personal and household products (e.g., supermarket, drug store, department store). Encourage students to compare the price of competing consumer products by considering:
  - unit price
  - discounts and sale price
  - coupon promotions.
- Invite a guest speaker from Consumer and Corporate Affairs to discuss consumer issues related to advertising, product pricing or credit practices.

BANKING

BASIC RESOURCE CORRELATION

Mathbase 2
- Chapter 6: Saving Your Money

SUGGESTED ACTIVITIES

1. Discuss the use of various kinds of bank accounts. Compare different types of accounts by identifying their similarities and differences (see Resource 1: Using Banking Services Effectively).

   Invite guest speakers from local banking institutions to explain the features of various types of accounts available to customers. Compare the services offered by different banking institutions.

   Simulate a variety of relevant situations that require selection of a bank account and banking institution. Use a problem-solving process to select the type of account and institution most appropriate to personal goals and needs (see Problem Solving, "Using Strategies to Solve Problems").

   Ask students with bank accounts to share the features of these accounts with the class. Encourage other students to open bank accounts by discussing the services and benefits that may be provided.

2. Provide first-hand experience in the use of actual bank forms (e.g., deposit slips, withdrawal slips, cheques, cheque registers, bank statements). Obtain specimen forms from local banks and simulate situations that require students to use these forms.

   Students need to recognize the importance of being able to write a cheque in our "cashless" society. As students often find it difficult to write the amount of a cheque in word form, be prepared to spend time reinforcing place value concepts and spelling skills used in cheque writing. Display a list of frequently misspelled number words. Encourage students to write "void" across the face of all simulated cheques.

3. Demonstrate the use of the cheque register in keeping a record of deposits, withdrawals and cheques written. Discuss its value in planning expenditures/budgets, in avoiding overdrafts, and in checking the accuracy of bank statements.

   Investigate the use of integers in representing account balances that result in an "overdraft" (see Using a Math Lab, "Number Systems and Operations").

   Simulate situations that require students to use the cheque register and maintain an account balance. Encourage students to select computational procedures (i.e., paper-and-pencil, calculator) appropriate to each situation that requires a calculation to be made.

   Emphasize/model appropriate strategies for checking the results of computation. Students may benefit from direct instruction in the use of mental arithmetic and estimation. Encourage students to share personal strategies they use in checking their work (see Computational Facility and Estimation, "Developing Strategies for Mental Arithmetic and Estimation").

4. Make a bulletin board display on banking. The display might include:

   - copies of actual banking forms
   - a sample passbook
   - a description of various types of bank accounts
   - a list of services offered by the bank
   - current interest rates paid on money saved and money borrowed
   - a glossary of terms related to banking.

5. Discuss the concept of "interest" as "rent paid for the use of another person's money".

   Develop an understanding of "interest rate" (e.g., an interest rate of 5% means that $5 of interest is paid on each $100 that is borrowed or saved), and investigate/compare current interest rates for loans and savings plans.

   When calculating interest through the use of paper-and-pencil algorithms, select practical situations that involve simple number relationships. Assist students to develop strategies for finding interest through the use of the calculator and interest tables. Provide a variety of problems that require students to:

   - use the calculator in determining yearly interest on money borrowed/saved (e.g., $I = p \times r$)
   - use an interest table in determining interest for periods other than a year.

   Coach students in appropriate strategies for using the percent function on the calculator when calculating interest. Be aware, however, that the function of the percent key is not the same on all calculators (see Use of Technology, "The Calculator").

6. Assist students to write a computer program that will calculate interest. A list of commands in BASIC computer language are provided in Use of Technology, "The Computer".

   Ask students to examine (and possibly run) the following Basic program that is intended to calculate interest. Identify and correct errors in the program.

```
10   REM CALCULATES YEARLY SIMPLE INTEREST
20   PRINT "WHAT IS THE YEARLY INTEREST"
30   PRINT "RATE?"
40   INPUT R
50   PRINT "WHAT IS THE PRINCIPAL?"
60   INPUT P
70   PRINT "THE INTEREST IS:"
80   PRINT R*P
90   END
```

GETTING VALUE FOR MONEY SPENT

BASIC RESOURCE CORRELATION

Mathbase 1

- Chapter 8: Working With Percent
  Using Percent: Sales Tax
  Using Percent: Discount
  Using Percent: Discount and Sales Tax

COMPUTER SOFTWARE

Fast Facts

SUGGESTED ACTIVITIES

1. Simulate consumer situations in which students are required to:

   - estimate/calculate the total cost of a number of similar/dissimilar items they wish to purchase
   - make correct change for items purchased when amount tendered is less than $100.

   Cooperative planning with teachers in the practical arts may establish situations where students can use a cash register, count money and make change in real life contexts (e.g., the school cafeteria or school store).

2. Encourage students to actively investigate and compare various brands of consumer items.

   - Collect a variety of newspaper advertisements for items that students might wish to purchase:
     - underline key words and phrases that the consumer should be aware of in the advertisement
     - pose questions about the items that should be answered prior to making a purchase decision.

   - Provide students with a grocery list containing items advertised in the newspaper on a particular day. Have students go shopping through the grocery ads, recording item prices and determining the total cost of all items on the list. Encourage students to compare and evaluate their purchase decisions by asking question such as:
     - who had the lowest grocery bill? The largest grocery bill? Why?
     - how do prices charged by large chain stores, independent stores and discount stores compare?
     - what strategies did you use in shopping that were effective in reducing your food cost? (e.g., Did you use discount coupons? Did you select sale items?)

   - Ask students to select an item they wish to purchase (e.g., tape player, bicycle). Using catalogues and newspapers, gather information on five or six different brands/makes of this item. Compare information about each brand by making a table similar to the one below.

Brand	Size	Features	Quality	Warranty
Brand A				
Brand B				
Brand C				

   Draw a graph that depicts price variation according to brand/make.

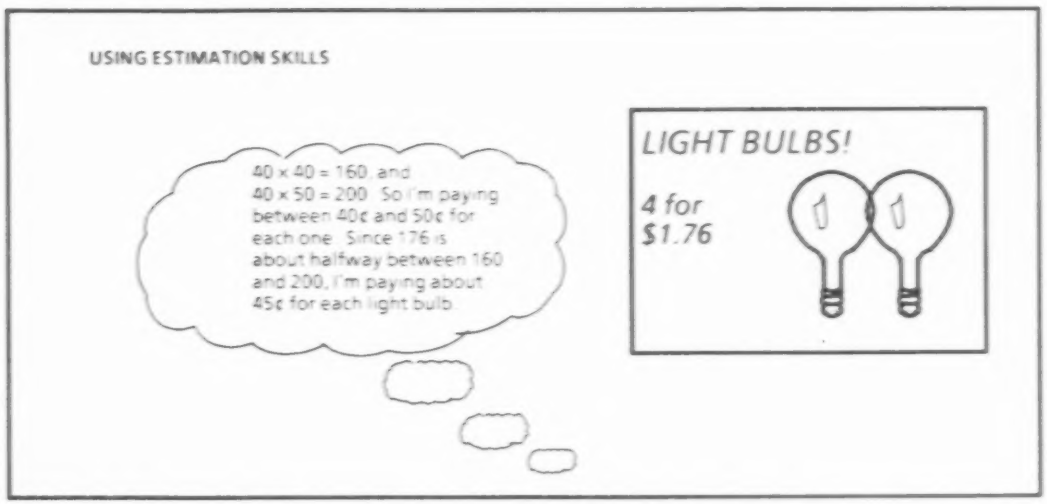
3. Ask each student to select a recipe from the food section of the local newspaper. Make a list of all ingredients required to make the recipe. Visit a local supermarket, recording brand names, sizes and prices of food items that must be purchased in order to make the recipe. Encourage students to compare results and to share strategies they used in making appropriate consumer decisions.

4. Provide frequent opportunities for students to practise skills in estimation and mental arithmetic when determining:

   - the cost of one article, given the cost of several
     e.g., "4 for $.79"
     How much will one cost?
   - the cost of several identical or different items, given individual costs
     e.g., "$1.59 each"
     How much will four cost?
     OR
     "How much will two at $1.09 and one at $.63 cost?"
   - the amount of change due when paying for a purchase with cash.

   Encourage students to find approximate/exact answers in consumer situations similar to these without the use of paper and pencil. Appropriate strategies in mental arithmetic and estimation should be modelled and shared by both teacher and student (see Computational Facility and Estimation, "Developing Strategies for Mental Arithmetic and Estimation").

CLARIFICATION/EXAMPLE


USING ESTIMATION SKILLS
40 x 40 = 160, and
40 x 50 = 200. So I'm paying
between 40¢ and 50¢ for
each one. Since 176 is
about halfway between 160
and 200, I'm paying about
45¢ for each light bulb.
LIGHT BULBS!
4 for
$1.76

5. Activities in "unit pricing" should enable students to recognize that:

   - unit price indicates how much the item would cost in a standard measurement (e.g., price per gram, per litre or per centimetre)
   - unit price enables the consumer to compare the price of competing brands of a product when sold in different sizes.

   Unit pricing will require development/review of the following concepts and skills:

- **Division**
  While computations in most problem-solving situations should be done using the calculator, unit pricing provides opportunity for the review of specific concepts and skills used in the division process (see Computational Facility and Estimation, "Developing Computational Process"). Be alert to difficulties students experience in:
  - placing the decimal point
  - using zero as a place holder in the quotient
  - rounding quotients to the nearest hundredth of a dollar (i.e., nearest cent).

- **Ratio and Proportion**
  Students should recognize the use of ratio in describing pricing rates that involve different units.

  e.g., $\frac{\text{(dollars)}}{\text{(grams)}}$ $\frac{\$\,95}{250\,g}$

  Equivalent ratios (proportions) can then be written to describe the process of unit pricing.

  e.g., $\frac{\$\,95}{250\,g} = \frac{?}{100\,g}$

  Students should use the common factor/multiple method of finding the missing component in proportions (see Using a Math Lab, "Ratio, Proportion and Percent").

  Encourage students to use the calculator when solving ratio and proportion problems (see Use of Technology, "The Calculator"). Calculator activities should emphasize:
  - the sequence of operations used in solving a proportion
  - the interpretation of answers displayed on the calculator
  - strategies for checking the reasonableness of results.

- **Measuring and Converting Units**
  Unit pricing often requires a familiarity with common units of length/capacity/mass, and ability to convert between frequently used units (see Using a Math Lab, "Geometry and Measurement"). Provide first-hand experience in the use of common units by displaying a variety of familiar containers/items, and asking students to:
  - estimate their length, capacity or mass
  - measure their length, capacity or mass as a check against estimates.

  Ability to convert among units of measure requires that students understand:
  - the meaning of metric prefixes (e.g., milli, centi, kilo)
  - strategies for multiplying and dividing numbers by 10 and multiples of 10.

6. Review the concept of percent by examining newspaper advertisements that display percent discounts.

   Students may benefit from working with pictorial representations of percent using the 10 × 10 grid. Shade the percent of discount in red. Look at the unshaded portion of the grid in order to determine what percent the discounted price is of the original price (see Using a Math Lab, "Ratio, Proportion and Percent").

   Encourage students to recall frequently used fraction, decimal and percent equivalents (e.g., one-half, quarters, tenths). Display pictures, diagrams and charts that illustrate the equivalents that students must use.

7. Ask students to calculate the sale price of articles advertised with percent discounts. Although discounts might be calculated using the proportion, decimal or fraction method, students will likely experience most success using the decimal method.

Example: An article is on sale at a 25% discount.
The original price was $8.00.
25% of $8.00
= 0.25 × $8.00
= $2.00

Discount = $2.00
Sale price = $8.00 − $2.00 = $6.00

Perform related computations on the calculator (see Use of Technology, "The Calculator"). Model and discuss strategies that can be used in checking the reasonableness of the discounts and sale prices that have been calculated.

CLARIFICATION/EXAMPLE

USING ESTIMATION SKILLS

Item for Sale	Price	Discount	Estimate of Savings
Tape recording	$8.88	10%	1/10 of $9 = $0.90
Calculator	$6.58	25%	1/4 of $6.40 = $1.60

8. Encourage students to recognize that stores sometimes use a sale to attract customers, and that the sale may not offer substantial benefits to the consumer. Provide opportunity for students to evaluate a current sale or promotion by considering:

- the number of sale items the store has to offer (i.e., there may be only a few items that are actually on sale in the store)
- the condition of the sale items (i.e., the sale items may be "seconds", or their shelf-life may have expired)
- the original price of sale items before they are discounted (i.e., the original price may be so high that even with the discount, the customer would not realize a saving).

9. Discuss the advantages/disadvantages of using credit and making installment purchases. Structure student thinking and discussion through use of the "PMI" strategy provided as Resource 2: PMI – Critical Thinking Tool. Invite a guest speaker from Consumer and Corporate Affairs to discuss important considerations to be made by the consumer before using credit.

Investigate various credit/installment plans offered by local companies by visiting stores in the community (e.g., appliance stores, car dealerships). Assist students to determine the "cost of credit" in real life situations by comparing the cash price of an item to the total cost when purchased through an installment plan or by taking a loan.

BUDGETING

BASIC RESOURCE CORRELATION

Mathbase 1
- Chapter 4: Data Graphs
  Displaying Data: Pictographs
  Displaying Data: Bar Graphs

COMPUTER SOFTWARE

Math Strategies: Problem Solving

SUGGESTED ACTIVITIES

1. Ask students to keep a record of their personal spending for a period of several weeks. Classify expenditures into common categories (e.g., food, clothing, recreation, entertainment, travel, gifts). Encourage students to become aware of how they spend their money.

   Students may wish to record their expenditures using forms similar to those provided in Resource 3: Expense Diary, and Resource 4: Monthly Record of Expenses.

2. Ask students to compare their income with their expenses. Summarize data collected using a table similar to the one illustrated below.

CLARIFICATION/EXAMPLE

Date	Details	Income	Expenditures
Oct. 10	Balance on hand	$20.25	
Oct. 10	Lunch		$ .95
Oct. 11	Cassette tape		7.35
Oct. 11	Lunch		.80
Oct. 12	Shirt		6.00
Oct. 12	Baby-sitting	7.00	
Oct. 14	Lunch		1.05
Oct. 16	Movie		5.00
	TOTALS	$27.25	$21.15

Review place value and addition/subtraction of whole numbers and decimals as required (see Computational Facility and Estimation, "Developing Computational Process"). If computational algorithms are understood by students, use calculators in summarizing income and expenses.

Emphasize the importance of rounding, estimation and mental arithmetic in checking the reasonableness of sums and differences that are obtained. Model appropriate techniques for checking results.

3. Discuss the fact that a significant part of personal expenditure is used for pleasure (wants) rather than absolute necessities (needs). In the budgeting process, one should begin by planning for necessary expenditures, and determine pleasure expenditures according to the balance.

   Encourage students to use appropriate problem-solving strategies in developing personal budgets (see Problem Solving, "Using Strategies to Solve Problems"). Ask students to identify:

   - personal wants and needs of a material nature
   - variables and constants in the budgeting process
   - potential problems in their personal spending patterns
   - strategies for resolving these problems
   - alternative patterns for personal spending.

4. Discuss the benefits of a regular saving plan in providing for:

   - major purchases (e.g., car, stereo)
   - future education
   - emergencies (e.g., illness, unemployment)
   - holidays.

   Activities similar to the one below will illustrate the effectiveness of a saving plan in providing for major purchases. Encourage students to "shop" for items of their choice in the newspaper, and devise appropriate savings plans that will permit the purchase of items selected.

   CLARIFICATION/EXAMPLE

Items you would like to buy	Cost	Amount you can save per month	Number of months you will have to save
A. _____	_____	$ 5.00	_____
B. _____	_____	$ 8.00	_____
C. _____	_____	$ 10.00	_____

5. Ask students to complete a budget showing anticipated income and expenditures for the next several weeks or months. Discuss the importance of "looking ahead" in order to plan for needs.

   Encourage students to recognize the value of a budget in realizing personal goals and ambitions. Review the three stages of problem solving and apply appropriate strategies to a variety of budgeting dilemmas that may be simulated or real.

RESOURCE 1: USING BANKING SERVICES EFFECTIVELY[1]

Banks serve two main purposes. They keep money for people, and they lend money to people. Banks also provide many other services that deal with money. Examples of these services are:

- keep money safe from theft
- transfer money from bank to bank
- change currencies (example: Canadian dollars to American dollars).

TYPES OF BANK ACCOUNTS

Most bank offer these accounts:

Regular (non-chequing) Savings Account

- a passbook is provided to customers as a receipt, updated as often as you wish
- only cash withdrawals are allowed (no cheques may be written)
- interest is calculated on the minimum (smallest) balance for the month and is paid every three or six months
- interest rates in most cases are higher than in a Daily Savings Account.

Daily Interest Savings Account

- usually a passbook (a record book for the customer) is provided
- only cash withdrawals are permitted (some banks allow a few cheques)
- interest is calculated on the balance at the end of the day
- interest is paid to your account at the end of every month, every three months or every six months (depending on the bank)
- interest rates are slightly lower than the Regular Savings Account rates. This is still a good account for the people whose savings account balance goes up and down during the month.
- some banks require a minimum balance to receive the interest rate quoted.

Daily Interest Chequing/Savings Account

- cheque and cash withdrawal privileges
- cheques are stored at the bank
- transactions are recorded in a personal passbook
- interest is calculated on each day's final balance and is paid at the end of each month
- no service charge for cash withdrawals.
- service charge for each cheque written.

[1] For permission to reprint copyrighted material, grateful acknowledgement is made to the following: Alberta Vocational Centre/Community Vocational Centres for the excerpts from *Mathematics: Applications and Information for the Adult Consumer*, 1986.

Personal Chequing Account

- a "demand" account (no waiting time imposed on any amount you want from your account)
- write cheques (no withdrawal slips used)
- record book with blank cheques provided
- statement with cancelled cheques mailed to customer
- no interest paid
- some banks have a service charge for each cheque written.

Term Deposits and Guaranteed Investment Certificates (GICs)

- savings accounts for a fixed length of time
- usually a minimum amount must be deposited
- terms can vary from one day to several years
- the rate of interest is fixed (does not change) and is often higher than the interest rates paid on regular savings accounts
- sometimes these accounts cannot be cashed-in until their final date. In other cases, early cashing-in can result in a penalty.

RESOURCE 2: PMI – CRITICAL THINKING TOOL

"Thinking is a skill, and like a skill, it can be developed and improved if one knows how."

– Edward deBono

There are many proponents of direct teaching of thinking as a skill and Edward de Bono is among the internationally recognized authorities in the field. He proposes a "tools method", whereby techniques for guiding the thinking processes are taught as discrete skills, practised in elementary contexts and later applied spontaneously and independently to real problems. The real life problems may change, but the tools to solve those problems remain constant.

Example: PMI tool. This tool reminds the thinker to first direct his or her attention to the **Plus** points, then the **Minus** points, and finally the **Interesting** points of a new idea. The thinker is encouraged to make an honest and thorough search in each direction to complete the thinking process relative to the problem.

Should I buy the stereo on an instalment plan now, or wait until I have saved enough to pay cash for it?

Plus	Minus	Interesting
• I will have the stereo now.	• It will cost more because of the interest.	• I will learn about buying on credit.
• I will have money left over to spend on other things.	• I will have the responsibility of paying a monthly bill.	• I will establish a credit rating.

Applied to real life problem-solving situations, a PMI can be done to clarify and help arrive at answers to such questions as:

- Should I complete my homework or go to the hockey game?
- Should I attend the dance knowing that mom will be unhappy with my decision?
- Should I lend my new sweater to my friend?

Plus	Minus	Interesting

REFERENCES

deBono, Edward. "Beyond Critical Thinking". Curriculum Review, January/February, 1986, pp. 13-16.

deBono, Edward. "The Practical Teaching of Thinking Using the CoRT Method", Special Services in the School, Vol. 3 No. 1/2, Fall/Winter 1986, pp. 33-47.

RESOURCE 3: EXPENSE DIARY[1]

Date	ITEM...WHAT? PURPOSE...WHY? WHERE? DESCRIPTION? FOR WHOM?	AMOUNT	

[1] For permission to reprint copyrighted material, grateful acknowledgement is made to the following: Alberta Consumer and Corporate Affairs for the excerpts from the *Expense Diary*, 1986

RESOURCE 4: MONTHLY RECORD OF EXPENSES

Days	Week 1		Week 2		Week 3		Week 4	
	Item	Amount	Item	Amount	Item	Amount	Item	Amount
Monday								
Tuesday								
Wednesday								
Thursday								
Friday								
Saturday								
Sunday								

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I.O.P. MATHEMATICS 9
WORLD OF WORK

SUB-THEMES

- USING MATH ON THE JOB
- WORKING WITH SCALE DRAWINGS
- PLANNING FOR PIECEWORK

RATIONALE

This theme provides students with opportunities to:

- develop skills required in performing clerical tasks associated with a variety of employment opportunities (e.g., stock person, cashier, waiter/waitress)
- apply number and measurement skills to practical situations involving the use of scale drawings and diagrams
- develop strategies for planning/designing a project that involves piecework payment.

Incorporating activities that relate to the part-time jobs students may presently have, or to employment opportunities currently available within the local community will ensure that learning experiences are relevant and meaningful. Thematic investigations should develop an awareness of entrepreneurial opportunities present in the local community.

Students will continue to develop and reinforce computational skills involving the use of whole numbers, decimals, fractions and percent. Calculators should be used on regular basis in performing many of the routine calculations. Measurement activities should place emphasis on estimation before application of actual skills in measurement. This approach will enable students to establish a "feel" for the size of various units, and enhance ability to select units that are appropriate to the task or problem.

Cooperative planning with other subject areas will facilitate a broader and more meaningful coverage of thematic objectives. The community offers additional resources for providing real life experience with many of the learning objectives.

Teachers are encouraged to reference the "Generic Strategies" section of this manual when planning for instruction. Strategies particularly relevant to the learning objectives and activities outlined in this theme can be found in:

- Problem Solving
- Use of Technology
- Computational Facility and Estimation
- Using a Math Lab.

THEMATIC OBJECTIVES

USING MATH ON THE JOB

- Develops a strategy for taking/maintaining inventory:
  - counts items
  - records stock on hand through the use of tables/charts
  - calculates differences in stock on hand at the beginning and end of a time period
  - calculates the total value of present inventory
  - recognizes the use of computers in inventory control.
- Develops skills used in cashiering:
  - recognizes functions performed by the cash register
  - recognizes the dollar value of rolls of coins
  - completes a bill of sale/charge slip
  - makes correct change for purchases when amount offered in tender is less than $100
  - completes a cash balance sheet.
- Develops clerical skills used when serving food in a restaurant or fast-food outlet:
  - identifies the cost of items on a menu
  - completes a customer order, calculating total cost of all items on the order
  - estimates/calculates an appropriate gratuity on the basis of the total cost of a food order/ quality of service/local community practices.

WORKING WITH SCALE DRAWINGS

- Recognizes/constructs basic one-, two- and three-dimensional figures.
- Constructs geometric patterns/designs using a variety of tools and instruments.
- Enlarges/reduces geometric figures and designs using dot paper/grids/geoboards.
- Interprets scale drawings (e.g., floor plans, maps, technical drawings).
- Constructs scale drawings/models of real objects or projects.

PLANNING FOR PIECEWORK

- Develops an understanding of vocabulary related to entrepreneurial activities (e.g., fixed costs, overhead, profit, selling price).
- Identifies projects that might provide piecework earnings (e.g., construction work, service jobs).
- Develops a plan for a piecework project:
  - estimates/calculates quantity and cost of materials required for the project
  - estimates amount of time required to complete the project and potential earnings based upon an appropriate hourly wage
  - determines the total cost of the project including materials and labour.
- Completes the project and evaluates success:
  - compares estimated cost of materials to actual cost of materials
  - compares estimated time to complete the project to actual working time
  - compares estimated profit to actual profit

CONTEXT FOR INSTRUCTION

PROBLEM SOLVING

- Develops appropriate strategies for counting change in situations that relate to buying or selling.
- Relates computation and measurement skills to situations involving scale diagrams and models.
- Uses a problem-solving strategy in planning and evaluating the success of an entrepreneurial activity (e.g., a piecework project).

USE OF TECHNOLOGY

- Uses the calculator and appropriate formulas in determining perimeter and area.
- Uses the calculator iteratively in monitoring inventory and determining quantity/cost of materials required for a project.
- Uses appropriate measuring tools/meters/gauges.
- Uses computer programs in the construction of geometric figures and patterns.
- Reads tables and charts as required to determine materials and sequence of operations appropriate to a given task.
- Uses computer programs as required to develop/reinforce number skills and problem-solving strategies.

COMPUTATIONAL FACILITY AND ESTIMATION

- Makes estimates of length/perimeter/area/mass/capacity/time as required:
  - determines quantity of materials/time required for a project
  - checks the results of actual measurement.
- Reviews/maintains basic computational algorithms.
- Performs computations with the calculator on a regular basis.
- Uses estimation to check the results of computation in practical situations.
- Uses mental arithmetic where appropriate to expedite solutions to quantitative problems.

SUPPORTING CONCEPTS, SKILLS AND ATTITUDES

Number Systems and Operations	Ratio, Proportion and Percent	Geometry and Measurement	Data Interpretation and Display	Algebra

NUMBER SYSTEMS AND OPERATIONS

- Performs computations with whole numbers, decimals and fractions using mental arithmetic/ paper-and-pencil algorithms/calculator.
- Rounds whole numbers and decimals as required.
- Rounds amounts of money to the nearest cent/dollar.
- Reads numbers represented on various scales and measuring devices.
- Determines "mental exact" solutions when multiplying or dividing by 10, 100 and 1000.
- Applies rules for the order of operations.

RATIO, PROPORTION AND PERCENT

- Uses ratio to represent the comparison of two quantities.
- Recognizes proportions as statements about equivalent ratios.
- Describes practical problem situations by writing proportions.
- Determines the value of the missing component in a proportion using the common factor/multiple method.
- Recognizes the concept of percent as a ratio indicating parts out of 100.
- Converts whole number percents to ratios/decimals and vice versa.
- Calculates/estimates a percent of a number in relevant applications.

GEOMETRY AND MEASUREMENT

- Identifies/constructs basic one-, two- and three-dimensional geometric figures.
- Recognizes common metric units of:
  - length (mm, cm, m, km)
  - mass (g, kg)
  - capacity (mL, L).
- Estimates and measures length/mass/capacity, selecting metric units and tools appropriate to the situation.
- Converts measurements as required among commonly used units.
- Estimates/measures perimeter and area.
- Estimates/measures/records time on a 12-hour and 24-hour clock.
- Adds/subtracts hours and minutes.
- Uses angle measures in the construction of geometric figures/patterns.

DATA INTERPRETATION AND DISPLAY

- Reads and interprets data presented in table/chart/graph form.
- Collects and records data in table/chart form.
- Makes inferences based on statistical data.

ALGEBRA

- Describes practical situations through the use of linear equations/formulas.
- Uses a formula to find perimeter and area.
- Uses variables to represent the relationship between cost price, selling price and profit.

INTEGRATION ACTIVITIES

LANGUAGE ARTS

- Uses reporting and note-taking skills in completing inventories/bills of sale/customer orders.
- Uses research skills when investigating entrepreneurial opportunities within the local community.

SCIENCE

- Estimates/measures length, mass, capacity and time when collecting data for scientific investigations.
- Compares measuring devices used in the science laboratory (e.g., graduated cylinders, balance scales) with those used in the workplace.

SOCIAL STUDIES

- Recognizes that entrepreneurial opportunities often provide for piecework employment.
- Investigates entrepreneurial opportunities within the local community.
- Recognizes the role of piecework in the Industrial Revolution.

PRACTICAL ARTS

- Estimates/measures length, perimeter, area, mass, capacity, and time in work-related situations.
- Uses appropriate measuring devices/gauges.
- Maintains an inventory of materials/supplies.
- Develops clerical skills that are related to project work:
  - monitors time spent on a project
  - determines quantity/cost of materials used
  - completes invoices/bills of sale/customer orders.
- Interprets/constructs scale drawings.
- Interprets tables and charts in order to complete a procedure.
- Recognizes that entrepreneurial opportunities often provide for piecework employment.

COMMUNITY PARTNERSHIPS

- Visit local business/industry in order to observe:
  - scale drawings/technical drawings/models in use
  - the application of measurement procedures
  - meters, gauges and other technologies used in measurement
  - the use of computer systems on the job.

  Community contacts might include automotive/appliance repair shops, construction projects, food service industries and architectural/drafting firms.
- Make scale drawings/models of various facilities found in the community:
  - floor plan of a community recreational facility
  - technical drawing of a piece of furniture (e.g., park bench)
  - model of the community park.
- Estimate the material/time required to complete various construction or service jobs in the local community:
  - painting a building
  - fencing a park
  - mowing a lawn
  - washing windows
  - clearing snow.

  Discuss factors that will affect the number of hours required to complete each job, and potential earnings for each job.
- Visit a local warehouse where students can observe stock taking and inventory control.
- Invite guest speakers from local business and industry to discuss the use of geometry/measurement/scale drawings/clerical skills in their trade or profession:
  - carpenter/painter/carpet layer
  - architect/draftsperson/interior decorator
  - garment or drapery fabricator
  - food service operator.
- Interview local residents who are involved in piecework employment through construction projects or the services they provide. Discuss the strategies they use for determining material costs, overhead and profit.

USING MATH ON THE JOB

BASIC RESOURCE CORRELATION

Mathbase 1

- Chapter 3: Working With Numbers
  - Dividing by a 2-Digit Divisor
- Chapter 6: Working With Decimals
  - Multiplying a Decimal by a Decimal
- Chapter 8: Working With Percents
  - Rounding and Percents
  - More % Key

Mathbase 2

- Chapter 3: Service With a Smile

COMPUTER SOFTWARE

Fast Facts

SUGGESTED ACTIVITIES

1. Invite the school secretary/business manager (or someone from local business and industry) to discuss:

   - the need for maintaining inventories of equipment and supplies
   - strategies for recording stock on hand and determining total value of inventory.

2. Discuss the purpose of maintaining an inventory of personal assets within the home.

   Ask students to compile an inventory of their personal assets (e.g., clothing, sporting equipment, bicycle, books, personal items). Inventories should include the quantity and approximate value of each item listed. A form that might be used in completing a personal inventory is provided in Resource 1: Inventory of Personal Assets.

3. Provide opportunities for students to take an inventory of equipment and supplies found in various areas of the school (e.g., physical education department, science laboratory, construction shop). Once items have been listed and counted, students might determine the value of inventory by consulting catalogues.

4. Prepare a bulletin board display of different coin wrappers. The display should illustrate the number of coins and the dollar equivalent associated with each wrapper.

   Provide opportunities for students to wrap their own coin collections or coins obtained from the school store.

5. Examine a variety of actual sales slips/charge slips/restaurant cheques obtained from local business (see Resource 2: Restaurant Cheque). Simulate situations in which students complete sales slips/bills of sale. Coach students in appropriate procedures for numeric dating, for listing purchase items, and for determining individual/total costs.

   Encourage students to use mental arithmetic and the calculator to determine the individual and total cost of purchase items. Emphasize the use of estimation in checking the accuracy of computation (see Computational Facility and Estimation, "Developing Strategies for Estimation").

6. Provide opportunity for students to practise counting change using both the "additive" and "subtractive" methods. Ask students to note situations in real life where each method is used.

   CLARIFICATION/EXAMPLE

   The total purchase is $23.49 and the amount given to the cashier is $30. Change could be given using the additive method or subtractive method.

   Additive Method:

Statement	Currency
$23.49	
$23.50	penny
$24.00	2 quarters
$25.00	$1 coin or $1 bill
$30.00	$5 bill

   Subtractive Method:

Statement	Currency
$6.51 change	
$5.00	$5 bill
$6.00	$1 coin or $1 bill
$6.50	2 quarters
$6.51	1 penny

7. Discuss the use of a "cash balance sheet" (see Resource 3: Cash Balance Sheet). Cooperative conferencing with teachers in the practical arts courses may identify situations in which the cash balance sheet is used within the school. Provide opportunities for students to count money and complete a cash balance sheet in simulated (or real) situations.

   Model strategies in mental arithmetic that might be used in completing this report (see Computational Facility and Estimation, "Developing Mental Arithmetic Skills"). Emphasize the use of estimation in checking the reasonableness and accuracy of results.

8. Discuss gratuities and the practice of "tipping" for service that is rendered. Identify factors that may determine the amount of a gratuity:
   - total cost of service rendered
   - quality of service
   - local community practices.

   Provide opportunities for students to estimate/calculate gratuities of 10%, 15% and 20% in simulated situations (see Using a Math Lab, "Ratio, Proportion and Percent"). Model strategies that will enable students to use mental arithmetic/estimation in determining appropriate gratuities for meals ordered in a restaurant.

WORKING WITH SCALE DRAWINGS

BASIC RESOURCE CORRELATION

Mathbase 1

- Chapter 3: Working With Numbers
  - Dividing by a Multiple of Ten
- Chapter 4: Data Graphs
  - Changing Scales on Graphs
- Chapter 6: Working With Decimals
  - Another Look at Multiplication
  - Using Your Skills
- Chapter 10: Geometry: Shapes and Designs
  - Scale Drawings
  - Distortions

SUGGESTED ACTIVITIES

SCALE DRAWINGS

1. Identify and discuss geometric figures present in everyday life. Investigate the use of geometric shapes and line relationships in community design and local architecture. Ask students to prepare a report on their findings, using pictures, sketches or diagrams.

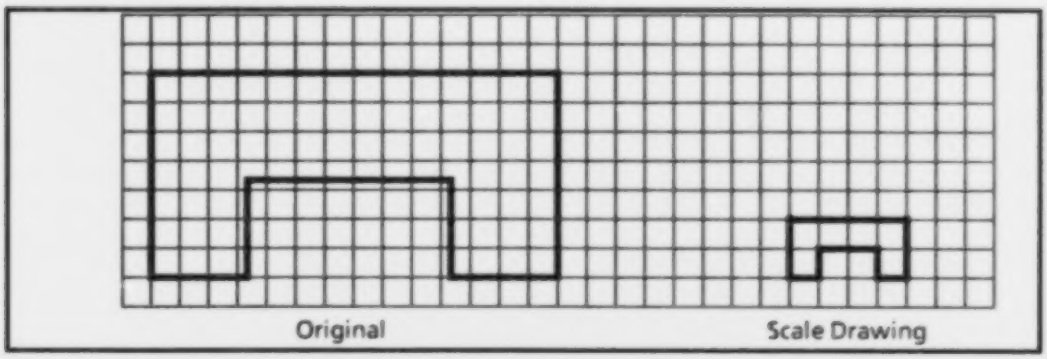
   Reinforce knowledge of basic two-dimensional figures by having students play "Attribute Dominoes" (see Using a Math Lab, "Geometry and Measurement").

2. Develop and reinforce geometric concepts/skills through projects that require students to construct geometric logos and patterns (see Using a Math Lab, "Geometry and Measurement"). Encourage students to be creative in the designs and patterns they produce. Increase motivation and effort by displaying student work in the classroom and hallway.

3. Use a LOGO computer program to produce geometric figures and designs (see Use of Technology, "The Computer"). Provide direction as to which line relationships/ geometric figures to incorporate into designs that are produced.

4. Introduce the concept of scale drawing by using dot paper/grid paper to enlarge or reduce simple geometric figures and designs.

CLARIFICATION/EXAMPLE

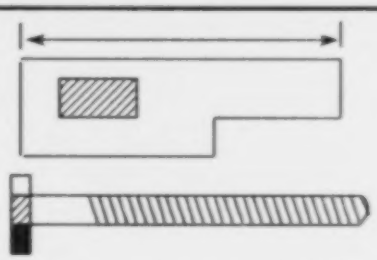


5. Discuss the use of ratio in describing the relationship between a drawing and the actual object. Students should recognize that the first number in the ratio represents the measure of the scale drawing, while the second number represents the measure of the actual object.

CLARIFICATION/EXAMPLE

If the scale is 1:60, what is the actual length of this pattern?

What is the actual length of this bolt if the scale is 2:1?



6. Review ratio and proportion skills that are used in solving scale problems. Students may benefit from a review of equivalent ratios using manipulatives and diagrams (see Using a Math Lab, "Ratio, Proportion and Percent").

Encourage students to "think through" the solution process, and to use the common factor/multiple method of solving related problems.

CLARIFICATION/EXAMPLE

A 150 metre pool is drawn to a scale of 1 cm = 50 m. How long is the pool on paper?

$$\frac{\text{cm}}{\text{m}} \qquad \frac{1}{50} \overset{\times 3}{\underset{\times 3}{}} = \frac{?}{150}$$

How many times longer than 50 m is the pool?
How many times longer than 1 cm should the diagram be?
What steps could be taken with the calculator to solve similar problems?

7. Provide opportunities for students to construct scale drawings and models.

   - Ask students to make a floor plan and furniture arrangement for one room in their house. Map out room dimensions, windows, doors, closets and furniture on centimetre grid paper. (A blackline master for producing centimetre grid paper is provided in the Grade 8 theme "Using Math at Home".)

   - Create a scale model of a public building or park in the community. Take actual measurements, determine an appropriate scale, and construct a scale model using cardboard or other suitable materials.

PLANNING FOR PIECEWORK

BASIC RESOURCE CORRELATION

Mathbase 1

- Chapter 1: Understanding Numbers
  - Order and Ranking
- Chapter 2: Solving Problems
  - Making an Organized List
- Chapter 12: Geometry Around Us
  - Angles
  - Measuring Angles
  - Angles Around Us
  - Drawing Angles
  - Circles

COMPUTER SOFTWARE

Math Strategies: Problem Solving

SUGGESTED ACTIVITIES

1. Through brainstorming, identify a variety of entrepreneurial opportunities in the local community that could provide students with piecework earnings.

   Examples:

Construction Jobs	Service Jobs
sewing/knitting	mowing lawns
painting	clearing snow
wood/metal projects	baby-sitting
pottery/ceramics	washing windows
building a fence	washing cars

2. Discuss the following ideas as they relate to the entrepreneurial opportunities identified by students:

   - cost price/selling price
   - fixed costs
   - overhead
   - labour costs
   - profit.

3. Provide opportunities for students to identify piecework projects that involve the use of perimeter and area skills. Projects might relate to those undertaken by students in the Practical Arts Program, or to other entrepreneurial activities in the local community. A look at sale flyers and newspaper advertisements will prompt students to identify materials sold by length and area.

Examples:	Perimeter	Area
	wall panelling	painting
	trimming	carpeting
	fencing	tiling

Simulate a variety of projects of this nature and ask students to determine the quantity and cost of materials required to complete each project. Encourage students to organize their thinking and plan their work by suggesting the following steps:

- take the required measurements
- interpret/construct scale drawings as required
- determine perimeter/area
- calculate quantity of material required
- calculate cost of materials.

Perform necessary computations using a calculator. Emphasize the correct order of operations, proper sequence for entering numbers into the calculator, and the interpretation of numbers displayed on the calculator (see Use of Technology, "The Calculator").

Encourage students to develop "referents" for the units of measure they use, and to use these referents in checking the reasonableness of the results they obtain.

4. Examine technical drawings/patterns that are used by students in the Practical Arts Program. (Conferencing with practical arts teachers will facilitate the identification of technical drawings and patterns used by students in work-related situations.)

Assist students to interpret these drawings/patterns, and to perform calculations that will determine the:

- quantity of materials used in each situation
- cost of materials used.

5. Ask each student to select a simple piecework project that will be completed through the use of facilities at school/home. Projects might relate to a practical arts course, or to a hobby in which the student has interest/experience.

Prepare plans for the project. Plans should include:

- a scale drawing of the project
- a list of materials that will be required
- the "cost price" of the finished project
- an estimate of the length of time required to complete the project
- a suggested "selling price" for the project that includes cost of materials and profit.

Upon completion, encourage students to evaluate the success of their project by:

- comparing estimated material costs to actual material costs
- comparing estimated working time to actual working time
- considering the appropriateness of the suggested selling price in light of marketing potential and actual working time.

(See Evaluation, "Interview Guide for Project Work" for suggested student assessment of this activity).

RESOURCE 1: INVENTORY OF PERSONAL ASSETS

ASSET	VALUE
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11.	
12.	
13.	
14.	
15.	
16.	
17.	
18.	
19.	
20.	
TOTAL	

RESOURCE 2: RESTAURANT CHEQUE

GREAT STEAK HOUSE FAMILY RESTAURANT				
SERVER	NO. OF PERSONS	TABLE NO.	DATE	
			AMOUNT	
FOOD SALES				
OTHER CHARGES				
TOTAL Thank you for eating at the GREAT STEAK HOUSE				

RESOURCE 3: CASH BALANCE SHEET

CASHIER _____ DATE _____

CASH RECEIPTS

Coins

Pennies	_____ × $.01 = _____
Nickels	_____ × .05 = _____
Dimes	_____ × .10 = _____
Quarters	_____ × .25 = _____
Half Dollars	_____ × .50 = _____
Dollar Coins	_____ × 1.00 = _____

Rolls

_____	_____ × _____ = _____
_____	_____ × _____ = _____

Bills

Ones	_____ × $ 1.00 = _____
Twos	_____ × 2.00 = _____
Fives	_____ × 5.00 = _____
Tens	_____ × 10.00 = _____
Twenties	_____ × 20.00 = _____
Fifties	_____ × 50.00 = _____
Hundreds	_____ × 100.00 = _____

Total Cash _____

Total Cash Sales _____

Difference _____

I.O.P. MATHEMATICS 9

USING MATH AT HOME

SUB-THEMES

- MAKING HOME IMPROVEMENTS
- PERSONAL NUTRITION AND GROWTH

RATIONALE

This theme provides opportunities for students to apply mathematical skills to familiar household situations. Many of the activities will reinforce concepts and skills that have been previously developed.

Measurement should be viewed as a strategy used to gather information that is required in problem-solving situations, and not as an activity in itself. Throughout this theme, measurement activities should be related to real life tasks, with emphasis on estimation before application of actual skills in measurement. This approach will enable the student to get a "feel" for the size of various units, and enhance student ability to select units that are appropriate to the task or problem.

Many opportunities exist for the integration of learning activities with other subject areas. Cooperative planning will be useful in planning projects that are relevant to students' needs in other areas of the program. The nature of the theme makes each student's home a community partnership, and each care-giver a community resource person. The teacher should take advantage of this in planning thematic activities.

Teachers are encouraged to reference the "Generic Strategies" section of this manual when planning for instruction. Strategies particularly relevant to the learning objectives and activities outlined in this theme can be found in:

- Problem Solving
- Use of Technology
- Using a Math Lab.

THEMATIC OBJECTIVES

MAKING HOME IMPROVEMENTS

- Estimates/measures household dimensions in mm, cm and m.
- Interprets/constructs scale drawings of a room that indicate:
  - room dimensions
  - windows/doors/closets
  - furniture size/location.
- Estimates/calculates perimeter and area in determining materials required for home construction/repair projects:
  - centimetres/metres of trim
  - square metres of floor covering
  - litres of paint
  - rolls of wallpaper.
- Determines the cost of materials used in redecorating a room:
  - floor covering
  - paint/wallpaper
  - tiles/panelling
  - trim.

PERSONAL NUTRITION AND GROWTH

- Monitors daily caloric intake.
- Compares personal eating habits with nutritional guidelines provided in the Canada Food Guide.
- Investigates the ingredients/nutritional content of familiar foods and recipes.
- Adjusts recipe ingredients (i.e., increases or decreases the recipe) according to need.
- Monitors personal growth:
  - measures body mass to nearest kg
  - measures height to nearest cm.
- Reads and interprets height/weight/age charts.
- Relates body size to appropriate clothing size.

CONTEXT FOR INSTRUCTION

PROBLEM SOLVING

- Uses a problem-solving strategy in determining the quantity/cost of materials required for a home construction/repair project.
- Recognizes the approximate nature of measurement and develops an ability to select appropriate units and tools for the task.
- Solves problems that require the application of measurement skills in real life situations.

USE OF TECHNOLOGY

- Uses the calculator and appropriate formulas in determining perimeter and area.
- Uses the calculator iteratively in determining quantity/cost of materials required for a project.
- Recognizes and uses appropriate measuring tools and devices.
- Reads tables and charts as required in completing tasks.
- Uses computer programs as required to reinforce number/measurement skills and develop problem-solving strategies.

COMPUTATIONAL FACILITY AND ESTIMATION

- Makes estimates of length, perimeter and area as required in order to:
  - determine quantity of materials required for a project
  - check the results of actual measurements.
- Reviews/maintains basic computational algorithms.
- Performs computations with the calculator on a regular basis.
- Uses estimation to check the results of computation in practical situations.
- Uses mental arithmetic where appropriate to expedite solutions to quantitative problems.

SUPPORTING CONCEPTS, SKILLS AND ATTITUDES

Number Systems and Operations	Ratio, Proportion and Percent	**Geometry and Measurement**	**Data Interpretation and Display**	**Algebra**

NUMBER SYSTEMS AND OPERATIONS

- Performs computations with whole numbers, decimals and fractions using mental arithmetic/ paper-and-pencil algorithms/calculator.
- Rounds whole numbers and decimals as required.
- Rounds amounts of money to the nearest cent/dollar.
- Determines "mental exact" solutions when multiplying or dividing by 10, 100 and 1000.
- Applies rules for the order of operations.
- Converts fractions into decimals and vice versa.
- Demonstrates addition, subtraction and multiplication of fractions.

RATIO, PROPORTION AND PERCENT

- Uses ratio to represent a comparison of two quantities.
- Recognizes proportions as statements about equivalent ratios.
- Describes practical problem situations by writing proportions.
- Determines the value of the missing component in a proportion using the common factor/multiple method.
- Recognizes the concept of percent as a ratio indicating parts out of 100.
- Converts whole number percents to ratios/decimals and vice versa.
- Calculates/estimates a percent of a number in relevant applications.

GEOMETRY AND MEASUREMENT

- Identifies/constructs basic one-, two- and three-dimensional geometric figures.
- Uses angle measure in the construction of geometric figures/patterns and scale drawings.
- Recognizes common metric units of:
  - length (mm, cm, m)
  - mass (g, kg)
  - capacity (mL, L).
- Estimates and measures length/mass/capacity, selecting units and tools appropriate to the situation.
- Converts measurements as required among commonly used units.
- Estimates/measures perimeter and area.

DATA INTERPRETATION AND DISPLAY

- Reads and interprets data presented in table/chart/graph form.
- Collects, records and displays data in table/chart/graph form.
- Interprets/calculates arithmetical average in practical situations.

ALGEBRA

- Describes practical situations through the use of variables/linear equations/formulas.
- Uses formulas to find perimeter and area.

INTEGRATION ACTIVITIES

LANGUAGE ARTS

- Uses reporting and note-taking skills in planning construction/repair projects in the home.
- Discusses the relationship between nutrition, health and self-concept.

SCIENCE

- Estimates/measures length, mass and capacity when collecting data for scientific investigation.
- Compares measuring devices used in the science laboratory (e.g., graduated cylinders, balance scales) with those used in the home.

PRACTICAL ARTS

- Estimates/measures length, perimeter, area, mass and capacity in work-related situations.
- Uses appropriate measuring devices/gauges.
- Maintains an inventory of materials/supplies and their costs in completing a project.
- Interprets/constructs scale drawings.
- Interprets tables and charts in order to complete a task.
- Increases and decreases recipes according to need.

COMMUNITY PARTNERSHIP OPPORTUNITIES

- Invite a local carpenter/drapery fabricator (or other craftsperson) to discuss strategies used in determining quantity/cost of materials required for various home improvement projects.
- Visit a local lumber/paint/wallpaper/floor-covering store. Ask store personnel to demonstrate appropriate procedures for determining the quantity of wood/paint/wallpaper/carpet/tile required for various decorating projects. Investigate the cost of these materials.
- Invite a doctor/nurse to discuss:
  - nutritional guidelines
  - growth patterns (e.g., height/weight/age relationships).
- Invite a home economics teacher/seamstress to demonstrate appropriate procedures for taking body measurements and determining clothing size.

MAKING HOME IMPROVEMENTS

BASIC RESOURCE CORRELATION

Mathbase 1
- Chapter 6: Working With Decimals
  - Area
  - Investigating Area: The Rectangle
  - Investigating Area: The Triangle
  - Area Around Us

COMPUTER SOFTWARE

Math Strategies: Problem Solving

SUGGESTED ACTIVITIES

1. Review estimation and measurement skills by providing a variety of puzzles/games that involve work with length, perimeter and area. Sample activities of this nature are provided in Resource 1: Perimeter and Area Puzzles, and in Resource 2: An Area Puzzle.

2. Students often experience difficulty in visualizing "area" and require tactile experience with the concept in order to become reasonable estimators. Ask students to construct a square centimetre and square metre. Through discussion, encourage students to recognize:

   - referents for the cm^2 and m^2
   - the number of cm^2 required to cover a m^2
   - familiar applications of the cm^2 and m^2 in projects undertaken within the home.

3. Assist students to distinguish between perimeter (distance around) and area (surface covered). Provide students with centimetre grid paper and ask them to:

   - draw a variety of geometric figures having given areas
   - determine the perimeter of each geometric figure they have drawn
   - share their results with other students in the class.

   Initiate a discussion based on the results of this activity by asking questions such as:

   - Which figure has the largest perimeter? Smallest perimeter?
   - Must two figures with the same area also have the same perimeter?
   - Must two figures with the same perimeter have the same area?

4. Provide opportunities for students to construct tables in which they compare the perimeter and area of geometric figures.

The table below has been constructed to summarize the results of an investigation on the perimeter and area of rectangles. As students complete the table, they should be encouraged to devise appropriate formulas for perimeter and area.

Length (m)	Width (m)	Perimeter (m)	Area (m^2)
5	2	14	10
6	3		
7	4		
ℓ	w	$2(\ell + w)$	ℓw

5. Examine scale drawings/floor plans for various objects/rooms in the home. Identify symbols that are used and the meaning of scale (i.e., ratio of pattern size to actual size). Assist students to construct a scale drawing of their bedroom (or some other room in the home) that illustrates room dimensions and locations of windows/doors/closets/ furniture.

6. Ask students to make plans for decorating a room in their home. Decorating plans may include:

   - paint/wallpaper
   - panelling/tiling
   - new floor covering
   - drapery/blinds
   - mouldings/trim
   - new furniture.

   Establish a budget for the decorating project. Using catalogues, determine the quantity/cost of materials that will be needed to complete the project. Select items that will ensure that total cost does not exceed the amount of the budget.

7. Ask students to design a tiling pattern/tessellation that will cover a floor of given dimensions (see Using a Math Lab, "Geometry and Measurement"). Coach students in designing their pattern by:

   - suggesting geometric shapes that might be used
   - illustrating a variety of tiling patterns
   - discussing a possible "scale" that might be used.

   After making necessary plans, construct the pattern using pieces of coloured paper.

8. Provide opportunity for students to design a tessellation using a LOGO computer program (see Use of Technology, "The Computer"). An example of a LOGO procedure designed to produce a square tessellation is provided as Resource 3: Producing a Tessellation With LOGO. This program can be easily modified to suit teacher/student preferences.

PERSONAL NUTRITION AND GROWTH

BASIC RESOURCE CORRELATION

Mathbase 1
- Chapter 4: Data Graphs
  - Drawing Bar Graphs
- Chapter 9: Fractions, Ratios, and Rates
  - Investigating Ratios

Mathbase 2
- Chapter 7: Keeping Fit

SUGGESTED ACTIVITIES

1. Ask students to monitor and record the foods they eat over a period of one week. Using a calorie chart, determine total daily caloric intake, weekly intake, and average daily intake. Compare results with averages recommended for individuals of similar height/weight/age. Display the results of this investigation in graph form.

2. Compare personal eating habits with nutritional guidelines recommended in the "Canada Food Guide" (see Resource 4: Canada Food Guide).

   Invite a doctor/nurse/home economist to discuss:

   - the nutritional value of favourite "fast foods"
   - the effects of excess intake of carbohydrates and fats
   - strategies for planning a nutritionally sound diet.

3. Develop familiarity with various mass/capacity measures used in the *Canada Food Guide* through first-hand experience in measuring foods using the units. Demonstrate appropriate procedures for weighing food products using the spring scale and balance scale.

4. Provide opportunity for students to evaluate the nutritional content of favourite snack foods by reading ingredient labels. Note that ingredients are listed in order of quantity. If possible, determine what fraction of the product's mass (or volume) is represented by each ingredient.

5. Ask students to bring favourite recipes from home. Discuss the nutritional content of each recipe. Ask students to adjust each recipe to provide for two servings. Collect the recipes and make a cookbook entitled "Cooking for Two". If a food laboratory is available, provide opportunity for students to refine their measurement skills by making several of the recipes.

6. Have students determine the validity of the following statements by measuring themselves:

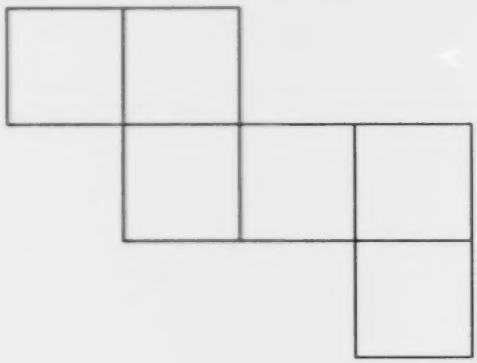
   - the length of the foot is the same as the distance from the inside of the wrist to the elbow
   - the arm span measured from finger tip to finger tip is the same as a person's height
   - the length of a person's foot is the same as the circumference of their head
   - the circumference of the head is 1/8 of a person's body height.

7. Provide opportunity for students to measure their own height and weight. Record results on a coded class list that ensures the confidentiality of personal data. Use arithmetical processes to determine the class average. Compare individual results/class averages with national averages for similar age groups. Information may be obtained from the district home economist.

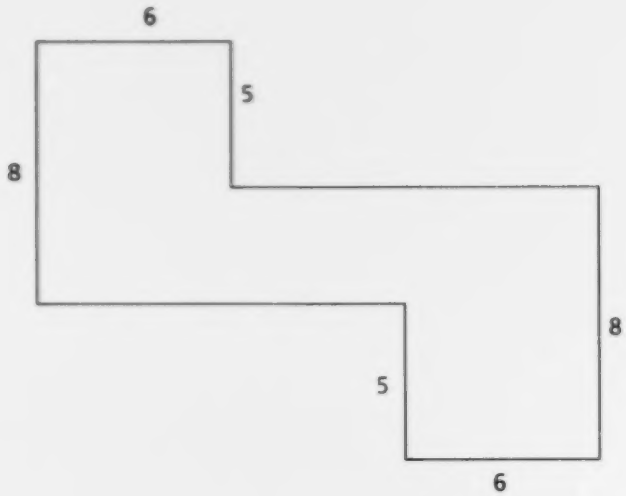
8. Relate personal body measurements (e.g., height, waist size, length of inseam, length of arm, neck size) to standard clothing sizes. Invite a home economics teacher/seamstress to discuss the use of these measurements in adjusting sewing patterns and selecting personal garments.

RESOURCE 1: PERIMETER AND AREA PUZZLES[1]

This figure consists of six congruent squares and has a total area of 294 cm^2. Find the perimeter of the figure.



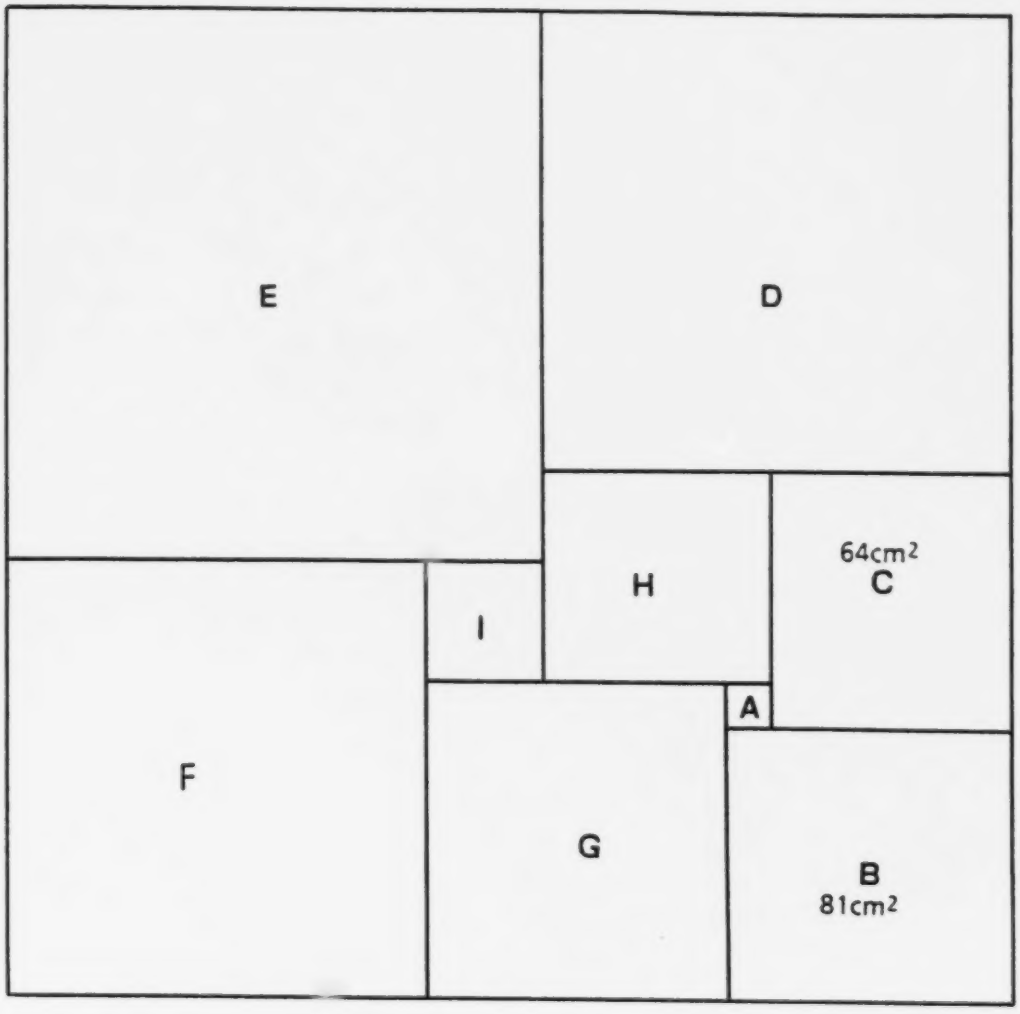
This figure has an area of 108 square units. What is the perimeter of the figure?



[1] Alberta Education. *Problem-Solving Challenge for Mathematics*, p. 44, 1985.

RESOURCE 2: AN AREA PUZZLE[1]

Using the information provided, find the area of each square region in the diagram below.



Answer: A – 1 cm² G – 100 cm²
D – 225 cm² H – 49 cm²
E – 324 cm² I – 16 cm²
F – 196 cm²

[1] Alberta Education. *Let Problem Solving Be the Focus in the 1980's*, p. 112, 1983.

RESOURCE 3: PRODUCING A TESSELLATION WITH LOGO

The following LOGO procedure will produce a square tessellation. The procedure is easily modified to suit individual preferences.

I. Make the basic shape:
(e.g., SQUARE 10)

```
TO SQUARE : S
   REPEAT 4[FD : S RT 90]
END
```

screen →

II. Make a pattern:

```
TO THREE_SQUARE
SQUARE 10
SQUARE 20
SQUARE 30
END
```

screen →

III. Start the pattern in the upper left hand corner:

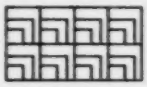
```
TO START
PU
LT 90
FD 120
RT 90
FD 80
PD
END
```

screen →

IV. Make a row of the pattern:

```
TO ROW.SQUARE
REPEAT 8 [THREE.SQUARE PU
RT 90 FD 30 LT 90 PD]
END
```

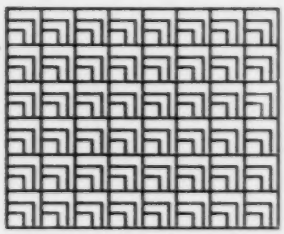
screen →



V. Make the tessellation:

```
TO TESSELLATE.SQUARES
START
REPEAT 6[ROW.SQUARE PU
LT 90 FD 240 RT 90 BK.30
PD]
END
```

screen →



RESOURCE 4: CANADA FOOD GUIDE[1]

Canada's Food Guide
Eat a variety of foods from each group every day
milk and milk products
Children up to 11 years 2-3 servings
Adolescents 3-4 servings
Pregnant and nursing women 3-4 servings
Adults 2 servings
meat, fish, poultry and alternates 2 servings
breads and cereals 3-5 servings
whole grain or enriched
fruits and vegetables 4-5 servings
Include at least two vegetables
Health and Welfare Canada
Santé et Bien-être social Canada
© Minister of Supply and Services Canada 1983
Cat. No. H58-32-1983-2E
Canada

[1] *Canada's Food Guide*, Health and Welfare Canada, 1983 and reproduced with permission of the Minister of Supply and Services Canada.

Canada's Food Guide

Variety

Choose different kinds of foods from within each group in appropriate numbers of servings and portion sizes

Energy Balance

Needs vary with age, sex and activity. Balance energy intake from foods with energy output from physical activity to control weight. Foods selected according to the Guide can supply 4000 – 6000 kJ (kilojoules) (1000 – 1400 kilocalories). For additional energy, increase the number and size of servings from the various food groups and or add other foods

Moderation

Select and prepare foods with limited amounts of fat, sugar and salt. If alcohol is consumed, use limited amounts

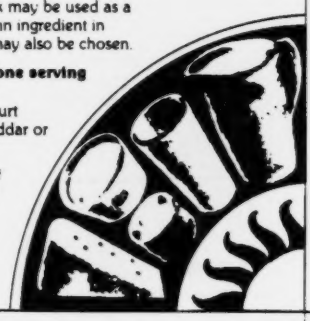
milk and milk products

Children up to 11 years	**2-3 servings**
Adolescents	**3-4 servings**
Pregnant and nursing women	**3-4 servings**
Adults	**2 servings**

Skim, 2%, whole, buttermilk, reconstituted dry or evaporated milk may be used as a beverage or as the main ingredient in other foods. Cheese may also be chosen.

Some examples of one serving
250 mL (1 cup) milk
175 mL (¾ cup) yoghurt
45 g (1½ ounces) cheddar or process cheese

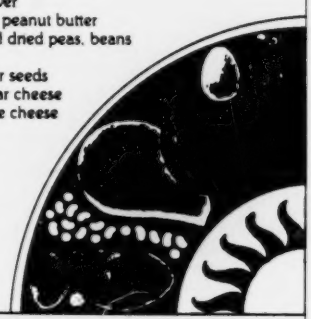
In addition, a supplement of vitamin D is recommended when milk is consumed which does not contain added vitamin D



meat, fish, poultry and alternates

2 servings

Some examples of one serving
60 to 90 g (2–3 ounces) cooked lean meat, fish, poultry or liver
60 mL (4 tablespoons) peanut butter
250 mL (1 cup) cooked dried peas, beans or lentils
125 mL (½ cup) nuts or seeds
60 g (2 ounces) cheddar cheese
125 mL (½ cup) cottage cheese
2 eggs

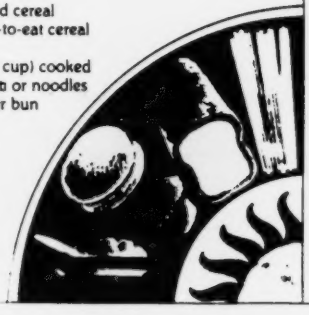


breads and cereals

3-5 servings

whole grain or enriched. Whole grain products are recommended.

Some examples of one serving
1 slice bread
125 mL (½ cup) cooked cereal
175 mL (¾ cup) ready-to-eat cereal
1 roll or muffin
125 to 175 mL (½ – ¾ cup) cooked rice, macaroni, spaghetti or noodles
½ hamburger or wiener bun



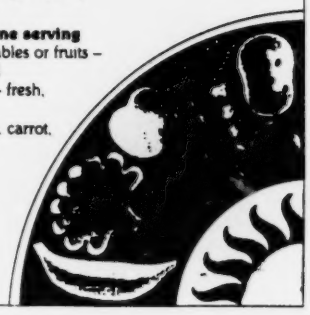
fruits and vegetables

4-5 servings

Include at least two vegetables.

Choose a variety of both vegetables and fruits — cooked, raw or their juices. Include yellow, green or green leafy vegetables

Some examples of one serving
125 mL (½ cup) vegetables or fruits – fresh, frozen or canned
125 mL (½ cup) juice – fresh, frozen or canned
1 medium-sized potato, carrot, tomato, peach, apple, orange or banana



I.O.P. MATHEMATICS 9
TRAVEL AND RECREATION

SUB-THEMES

- TRANSPORTATION
- PLANNING A TRIP
- SPORTS AND FITNESS

RATIONALE

Most students should be particularly interested in this theme as it involves use of their leisure time. Opportunities will be provided for students to investigate various aspects of the activities that frequently occupy their time spent out of school.

Transportation is a part of everyone's life, whether related to taking a bus or operating a vehicle. Planning a trip requires consideration of a variety of factors, and is an activity that each student will undertake at some point in their lives. Students frequently spend their leisure time in various sports and recreational activities, and will benefit from an understanding of the effect these activities have on personal health and fitness levels.

Teachers are encouraged to modify thematic learning objectives in meeting the needs of individual students and according to circumstances within the local community. Many opportunities exist throughout the theme for integrating mathematical instruction with activities undertaken in other subject areas (e.g., social studies, language arts). The local community contains a variety of resources that will provide students with real life experiences related to many of the learning objectives.

"Generic Strategies" that are particularly relevant to the learning objectives and activities outlined in this theme can be found in:

- Problem Solving
- Computational Facility and Estimation
- Using a Math Lab.

THEMATIC OBJECTIVES

TRANSPORTATION

- Recognizes line relationships/geometric shapes that are present in road signs.
- Reads and interprets bus/train/airline schedules.
- Compares the advantages/disadvantages of purchasing a bus pass with paying daily fare.
- Compares the cost of a taxi with the cost of public transportation.
- Recognizes the relationship between distance travelled, rate of travel, and travel time. Solves related problems.
- Estimates travel distances and travel time.
- Determines fuel economy for a vehicle as litres consumed per 100 kilometres (i.e., L/100km).
- Estimates/calculates the cost of fuel for travelling a given distance.

PLANNING A TRIP

- Plans a vacation. Selects:
  - a destination
  - method of transportation
  - type of accommodation
  - departure and return dates.
- Determines the approximate cost of accommodation and meals on a trip by considering:
  - number of persons travelling
  - destination
  - length of stay.
- Compares the estimated cost of car travel with the cost of bus/train/airplane travel.
- Estimates the cost of entertainment/excursions/gratuities on a trip.
- Makes an itinerary for a trip.
- Estimates the total cost of a trip/vacation.

SPORTS AND FITNESS

- Identifies energy components of a balanced diet. Analyzes personal diet in terms of energy components.
- Reads charts to determine recommended daily caloric intake and number of calories burned through various activities.
- Recognizes the relationship between calories consumed (or burned) and weight increase or decrease.
- Calculates individual fitness measures:
  - pulse rate
  - jogging rate
  - heart beat recovery rate
  - lung capacity.
- Compares individual fitness measures with accepted norms.

CONTEXT FOR INSTRUCTION

PROBLEM SOLVING

- Considers the advantages/disadvantages of different modes of travel.
- Uses a problem-solving strategy in planning/budgeting for travel.
- Solves routine problems involving travel distance, rate of travel and travel time.
- Monitors personal fitness level. Identifies strategies that will maintain/improve personal level of health and fitness.

USE OF TECHNOLOGY

- Uses a calculator iteratively.
- Uses tables, charts and schedules.
- Uses the media to collect information related to travel, recreation and sport.
- Recognizes the use of computers in compiling statistics and schedules.

COMPUTATIONAL FACILITY AND ESTIMATION

- Develops/reinforces basic computational algorithms.
- Performs computations with a calculator on a regular basis.
- Uses mental arithmetic in determining travel costs/fitness measures.
- Rounds as appropriate during calculations.
- Uses estimation in comparing the cost of different travel methods.
- Estimates travel distances and travel times.
- Uses estimation in checking the reasonableness of computational results.

SUPPORTING CONCEPTS, ATTITUDES AND SKILLS

Number Systems and Operations	Ratio, Proportion and Percent	Geometry and Measurement	Data Interpretation and Display	Algebra

NUMBER SYSTEMS AND OPERATIONS

- Reads/writes/orders decimals to thousandths.
- Performs basic operations with whole numbers and decimals.
- Determines "mental exact" solutions when multiplying or dividing by multiples of 10.
- Applies rules for the order of operations.
- Compares/orders fractions in applications.
- Identifies and determines equivalent fractions.
- Recognizes and expresses fractions in simplest form.
- Demonstrates basic operations with fractions in practical situations.

RATIO, PROPORTION AND PERCENT

- Recognizes ratios as ordered pairs of numbers showing comparison.
- Generates equivalent ratios using single-digit whole number constants.
- Recognizes proportions as statements about equivalent ratios.

- Determines the missing component in a proportion using the common factor/multiple method.
- Demonstrates the concept of percent as a ratio indicating parts out of 100.
- Converts whole number percents to ratios/decimals and vice versa.
- Calculates/estimates a percent of a number in relevant applications.

GEOMETRY AND MEASUREMENT

- Identifies basic one-, two- and three-dimensional figures.
- Recognizes common metric units of length, mass and capacity.
- Estimates/measures length, mass and capacity, selecting units and tools appropriate to the situation.
- Estimates/calculates distance between locations on a road map.
- Estimates/measures/records time on a 12-hour and 24-hour clock.
- Uses National Standards for numeric dating.

DATA INTERPRETATION AND DISPLAY

- Reads and interprets information presented in lists/charts/tables/graphs.
- Collects and records data using frequency tables/tally sheets.
- Displays data using tables/charts/graphs.

ALGEBRA

- Distinguishes between the use of variables and constants in concrete situations.
- Interprets/uses equations that describe practical situations (e.g., d = rt).
- Uses formulas to calculate personal fitness measures.

INTEGRATION ACTIVITIES

LANGUAGE ARTS

- Uses research skills to gather data related to travel and personal fitness.
- Uses reporting and note-taking skills to summarize data that has been gathered.
- Uses decision-making skills in planning a trip.
- Investigates the use of leisure time in the theme "How are you today?".
- Discusses the importance of personal fitness in the theme "Growing Today and Tomorrow".

SCIENCE

- Estimates/measures length, mass and capacity when collecting data for scientific investigation.
- Recognizes the impact of technology on the travel industry and levels of personal fitness/health.

SOCIAL STUDIES

- Uses city maps and Alberta road maps.
- Recognizes possible travel destinations within Alberta.
- Calculates/estimates distances on a road map.
- Investigates local recreational and fitness facilities in the theme "Community Awareness".

PRACTICAL ARTS

- Identifies potential travel destinations and recreational facilities in the theme "Smile You're a Tourist Attraction!".
- Uses skills from business education in preparing an itinerary for a trip.
- Recognizes the influence of fitness and health on personal development.

COMMUNITY PARTNERSHIP OPPORTUNITIES

- Visit/write a local tourist bureau or travel agency and request travel information (e.g., road maps, brochures describing potential travel destinations, travel costs).
- Visit a local bus/train/airline terminal. Request travel schedules, information on fares, and the cost of various travel packages.
- Invite the school nurse to demonstrate procedures for taking pulse rate, and to discuss the relationship between heart beat recovery rate and fitness.
- Visit a local health and fitness centre. Investigate the conditioning principles underlying the use of exercise bicycles, rowing machines and weight equipment.
- Invite a physical education teacher/local fitness expert to discuss:
  - personal fitness measures
  - strategies for improving fitness.

TRANSPORTATION

BASIC RESOURCE CORRELATION

Mathbase 1
- Chapter 8: Fractions, Ratios, and Rates
  - Rate

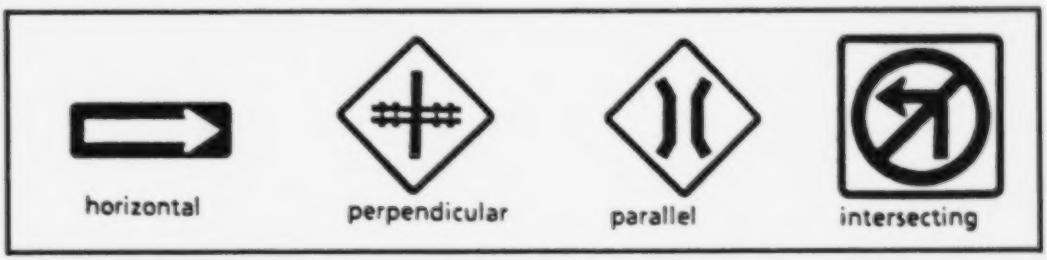
COMPUTER SOFTWARE

Fast Facts

SUGGESTED ACTIVITIES

1. Investigate the geometric shapes and line relationships present in road signs. Ask students to draw a variety of road signs and classify them according to shape. Identify various line relationships present in each road sign.

CLARIFICATION/EXAMPLE



2. Discuss the advantages/disadvantages of purchasing a bus pass compared with paying daily fare. Calculate the cost of a return bus trip to school each day for a period of one month. Compare this cost to the price of a monthly bus pass. Identify other factors that should be considered in deciding whether to purchase a pass or pay daily fare.

   Estimate the taxi fare for a trip within the community. Compare the taxi fare with bus fare for the same trip. Considering the difference in cost, what circumstances would suggest that transportation be provided by taxi rather than bus.

3. Provide opportunity for students to investigate the relationship between distance travelled, rate of travel and travel time.

   Develop an initial understanding of concepts by working with simple numerical relationships (e.g., How long would it take you to talk a distance of 10 km at a travel rate of 2 km/h?). Once concepts and relationships are understood, extend activities to more difficult situations (e.g., If you walk twice as fast, what effect will this have on the time taken to walk 10 km?). Encourage students to generate formulas that describe the relationships they discover (e.g., $d = rt$, $t = d/r$, $r = d/t$).

   Discuss the use of appropriate strategies for solving problems involving distance, rate and time (see Problem Solving, "Using Strategies to Solve Problems").

4. Provide students with a road map. For selected destinations, both near and far, ask students to:

   - estimate travel distance. Students may wish to use "referents" or "chunking" in making their estimates of distance (see Using a Math Lab, "Geometry and Measurement")
   - check the reasonableness of their estimates of distance by reading distance charts or using map scale to calculate distance
   - estimate travel time. Estimates should be based on a reasonable rate of travel for the method of transportation being considered
   - check the reasonableness of their estimates of time through calculations performed on the calculator.

5. Ask students to run the following computer program written in BASIC language. The program is designed to determine travel time for given distances and travel rates.

```
 10 REM FIND TIME OF TRIP
 20 PRINT " WHAT DISTANCE DO YOU WISH"
 25 PRINT "TO TRAVEL"
 30 INPUT X
 40 PRINT "AT WHAT RATE DO YOU WISH TO"
 45 PRINT "TRAVEL"
 50 INPUT Y
 60 PRINT "IT WILL TAKE YOU"
 70 LET Z = X/Y
 80 PRINT Z "HOURS"
 90 PRINT "DO YOU WISH TO FIND ANOTHER"
 95 PRINT "TIME?"
100 INPUT A$
110 IF A$ = "YES" THEN 20
120 PRINT "THANK YOU"
130 END
```

   After students have had the opportunity of running the program with several different inputs, ask them to suggest ways in which the program might be modified or improved. (For example, line 80 might be changed so that the answer is given in hours and minutes rather than in decimal hours.)

6. Discuss the use of ratio in describing fuel consumption (e.g., L/100 km). Investigate the average fuel consumption rates for several different vehicles. Using these rates, estimate/calculate fuel consumption/costs for various trips.

   Students may experience difficulty with ratio and proportion concepts. Appropriate strategies for developing these concepts are provided in Using a Math Lab, "Ratio, Proportion and Percent".

   Invite a local automotives expert to discuss topics related to fuel consumption.
   e.g., – strategies for monitoring/calculating fuel consumption
   – average fuel consumption rates for different types of vehicles
   – factors that may influence fuel consumption.

7. Select several travel destinations on an Alberta road map. Through the use of distance charts and average fuel consumption rates, ask students to determine:

   - travel distances
   - the approximate number of litres of fuel required to travel these distances
   - fuel costs for each trip (using current fuel prices)

PLANNING A TRIP

BASIC RESOURCE CORRELATION

Mathbase 1
- Chapter 13: Working With Integers
  - Introduction to Integers

Mathbase 2
- Chapter 2: Planning a Journey

COMPUTER SOFTWARE

Math Strategies: Problem Solving

SUGGESTED ACTIVITIES

1. Ask students (individually or in small groups) to plan a vacation. Vacation plans should include selection of:

   - a destination
   - a method of transportation
   - accommodation
   - departure and return dates.

   Prior to making these decisions, encourage students to visit/write a local tourist bureau or travel agency and request information on potential travel destinations. Information gathered may include:

   - road maps/travel brochures
   - travel packages that are available
   - fare schedules.

2. Invite a representative from a local travel agency to discuss factors to be considered in planning a trip. Discussion may relate to:

   - travel schedules and fares
   - the cost of accommodation and meals
   - other factors that may influence travel costs
   - how to make an itinerary.

3. Ask students to determine the approximate cost of transportation, accommodation and meals on the vacation they are planning.

   Discuss factors that may influence the cost of accommodation and meals:

   - number of persons travelling
   - destination
   - length of stay.

   Identify alternatives to motels/hotels/restaurants that may be acceptable, yet effective in reducing the cost of travel.

4. Provide opportunity for students to plan an itinerary for their vacation. The itinerary should include a description of each day's excursions/activities, along with an estimate of the costs. Ask students to use numeric dating and 24-hour time notation in preparing their itinerary.

5. Ask students to estimate the total cost of their vacation. Brainstorm in order to identify cost factors other than transportation, accommodation and meals that may need to be considered:

   - entertainment
   - gratuities
   - medical insurance.

6. Additional suggestions for planning a trip are provided in Using a Math Lab, "Project Work".

SPORTS AND FITNESS

BASIC RESOURCE CORRELATION

Mathbase 1

- Chapter 3: Working With Numbers
  - Using Your Skills
- Chapter 4: Data Graphs
  - Choosing a Scale
  - Interpreting Line Graphs
  - Practising Your Skills

SUGGESTED ACTIVITIES

1. Investigate the energy components of a balanced diet. Invite the school nurse to discuss nutritional guidelines, and the effect of personal eating habits on energy level.

   Ask students to monitor and record foods eaten for a period of several days/weeks. Have students analyze their personal diet in terms of:

   - caloric intake
   - energy components.

   Ask each student to display their daily caloric intake in the form of a bar/line graph.

   Identify strategies that might be used to maintain/increase present energy and fitness levels.

2. Investigate the number of calories burned in performing various physical activities (see Resource 1: Managing Weight Through Exercise). Using this information (or other information obtained from local health units/ fitness experts), estimate the number of calories burned each day while involved in normal activities.

3. Invite a local doctor/nurse to discuss the meaning of the following statement:

   "In order to lose 450 grams of body weight, you must burn approximately 3 500 calories."

Through discussion, encourage students to recognize that the statement is a generalization, and that other factors may cause personal circumstances to vary.

Ask the doctor/nurse to discuss the relationship between calories consumed (or burned) and weight increase or decrease. Provide opportunity for students to analyze their own eating/exercise habits, and to consider adjustments they may wish to make in order to increase/ decrease weight.

4. Have students monitor their pulse rate under varying conditions.

   Pulse rate is usually expressed as the number of heart beats per minute. Count heart beats for 30 seconds. Then multiply by two in order to determine the number of beats per minute.

5. Invite a physical education teacher/fitness expert to discuss factors that influence personal fitness:

   - frequency of exercise (e.g., how often?)
   - intensity of exercise (e.g., how hard?)
   - duration of exercise (e.g., for how long?).

   Ask the physical education teacher/fitness expert to demonstrate strategies used in determining individual fitness measures:

   - jogging rate
   - heart beat recovery rate
   - lung capacity.

6. Provide opportunities for students to measure their heart beat recovery rate in varying situations and activities. Compare personal rates to accepted norms.

   For example, after having determined resting heart rate, ask students to engage in 10 to 15 minutes of vigorous activity (e.g., a 10-minute run). Following the activity, monitor heart rate every minute until it returns to resting heart rate. Determine the number of minutes required for heart beat recovery. Compare results with other class members/accepted norms.

   Teachers should be aware of recommended "exertion" limits with respect to blood pressure/ pulse rate, and ensure that students do not exceed these limits in their investigations.

7. Tables and graphs similar to those provided in Resource 2: Monitoring Heart Rate might be used by students in monitoring their heart rates during interval exercises.

RESOURCE 1: MANAGING WEIGHT THROUGH EXERCISE[1]

Exercise is a factor in managing weight. In order to lose 450 grams, you must burn up approximately 3 500 calories. The chart indicates time that must be spent in various activities to burn 100 calories.

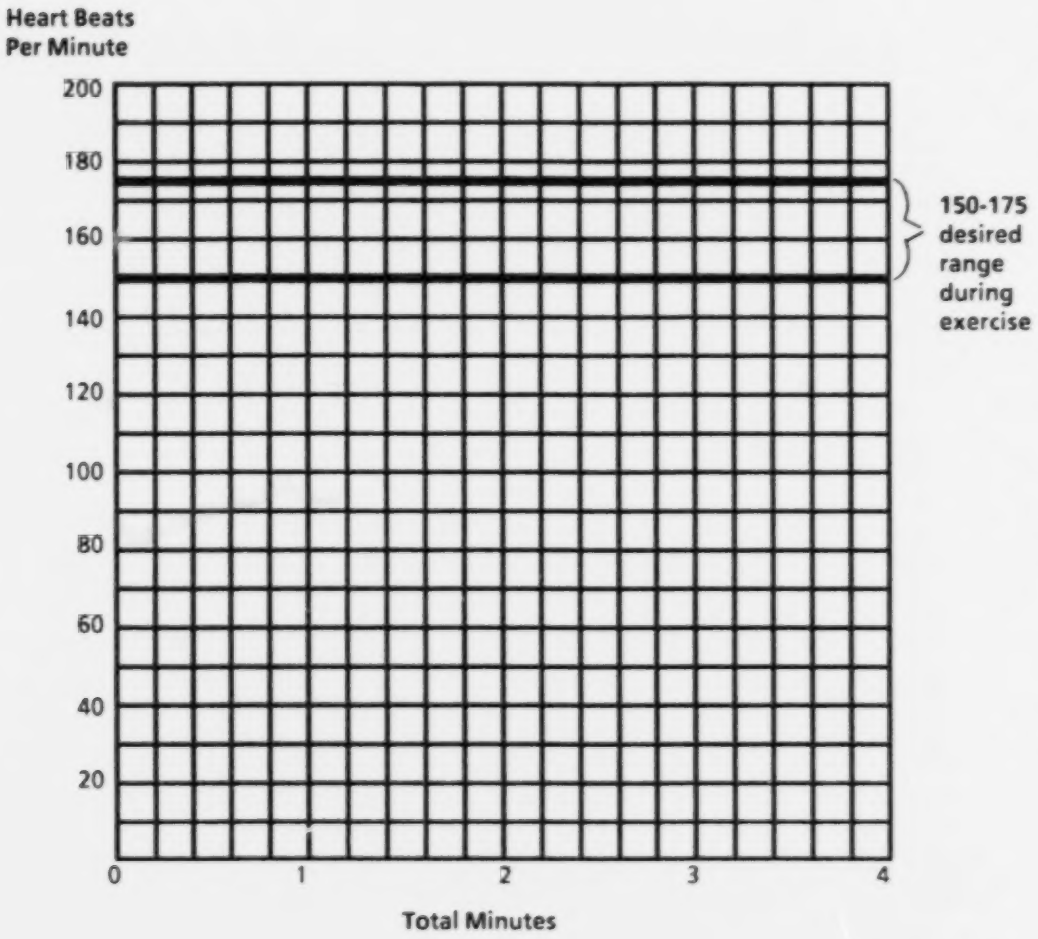
ACTIVITY	FEMALE 57 kg	MALE 73 kg
Clean windows	30 min	25 min
Garden	20 min	16 min
Calisthenics	20 min	19 min
Bicycle, 8 km/h	20 min	19 min
Bicycle, 20 km/h	13 min	9 min
Bowl (non-stop)	20 min	14 min
Ping-Pong	30 min	24 min
Run, 15 km/h	9 min	6 min
Run (in place)	5 min	4 min
Swim (crawl) 20 m/min	25 min	20 min
Tennis (moderately)	16 min	13 min
Dance (moderately)	30 min	23 min
Walk (fast pace)	19 min	14 min

[1] For permission to reprint copyrighted material, grateful acknowledgement is made to the following: Federated Co-operatives Limited for the excerpts from *Consumer's Guide, Co-op Consumer Counsellor*, p. F-5, 1983.

RESOURCE 2: MONITORING HEART RATE

HEART RATES DURING INTERVAL EXERCISES

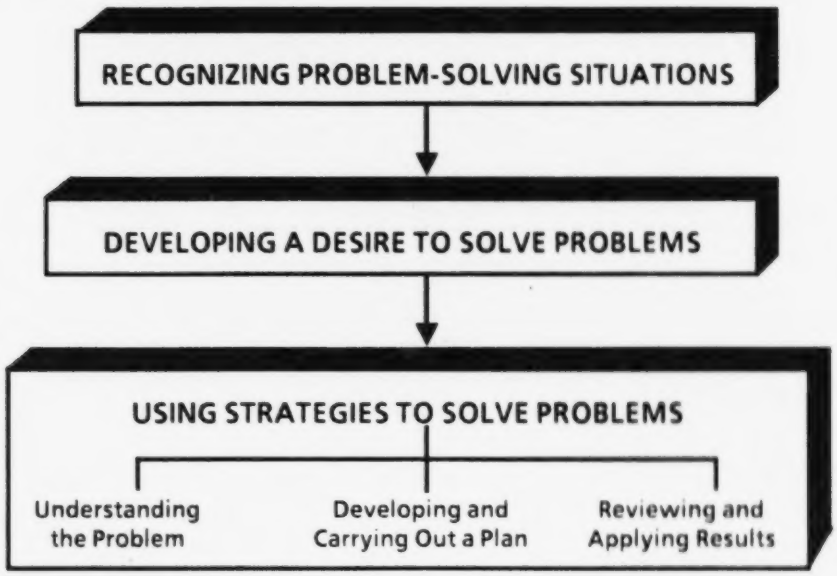
	REST	1 MINUTE	2 MINUTES	3 MINUTES	4 MINUTES
Trial 1					
Trial 2					
Trial 3					



PROBLEM SOLVING

Developing the ability to solve problems is probably the most important objective of the mathematics program. Advances in technology have required that changes occur in the focus of problem solving. Practice in finding answers to routine word problems by itself will not provide students with the problem-solving skills they will require in life. Instead, strategies for problem solving must be acquired that involve the use of critical and creative thinking skills. Students need to be provided with a variety of real life problems that are neither trivial nor predictable, and learn to solve these problems through the application of appropriate strategies and skills.

Problem solving should not be viewed as an isolated activity, but rather as a dynamic skill to be used throughout the mathematics curriculum. Appropriate attitudes and strategies for problem solving must be integrated throughout all themes and concept areas of the program. Teachers should become familiar with the framework for problem solving diagrammed below and described in the *Program of Studies/Curriculum Guide*.


RECOGNIZING PROBLEM-SOLVING SITUATIONS
DEVELOPING A DESIRE TO SOLVE PROBLEMS
USING STRATEGIES TO SOLVE PROBLEMS
Understanding the Problem
Developing and Carrying Out a Plan
Reviewing and Applying Results

A variety of instructional strategies that may be used in developing problem-solving ability are contained in this section of the manual including:

- Recognizing Problem-Solving Situations
- Developing a Desire to Solve Problems
- Using Strategies to Solve Problems
- Monitoring and Evaluating Progress
- Solving Problems in Cooperative Learning Situations

RECOGNIZING PROBLEM-SOLVING SITUATIONS

Problem solving must be viewed as encompassing more than finding answers to routine word problems. Students must realize that problem solving involves applying one's knowledge, skill and experience in any new and challenging situation. Encourage students to recognize situations at school, at home, at work and in the community that reflect aspects of problem solving.

CLARIFICATION/EXAMPLE

A problem may relate to any situation where:

- no readily apparent solution or means to a solution is evident
- a person can be temporarily perplexed
- there may be a single answer, many answers or no answer
- personal and societal factors are involved, as well as mathematical competencies.

Problem-solving situations should be both meaningful and interesting to students. Where possible, problems should be selected from a variety of real life situations that are relevant to students' experience. The problems selected should be challenging, yet solutions must be attainable to ensure that students experience success.

CLARIFICATION/EXAMPLE

A good problem should share some of the following characteristics:

- is relevant to student experience
- causes the learner to synthesize what has already been learned
- can be presented in more than one way
- can be solved in more than one way
- provides opportunities for verification of results
- does not mislead the student
- offers opportunities for extension and application

DEVELOPING A DESIRE TO SOLVE PROBLEMS

Student attitudes and beliefs about problem solving and about themselves can have a considerable influence on problem-solving performance. Students must learn to accept and appreciate that being perplexed and unsure is normal when first encountering a problem situation. Other attitudes and beliefs that foster the ability to solve problems include:

- "Problems can be solved in more than one way"
- "Problems may have more than one answer"
- "If the first strategy I try doesn't work, I'll try to find another strategy".

The teacher must ensure that the classroom atmosphere is supportive and encouraging, so that students are not afraid to take the risks associated with problem solving. As the peer group is extremely important to adolescents, care must be taken to avoid placing the student in potentially embarrassing situations. Avoid the use of descriptors such as "simple" or "easy" when describing a problem. Self-confidence may be weakened if a student cannot solve a problem that has been identified by the teacher as "easy".

CLARIFICATION/EXAMPLE

Strategies for developing appropriate attitudes and beliefs about problem solving:

- Create a positive classroom atmosphere that allows students to foster their own ideas and approaches to problem solving.
- Be supportive and encourage risk taking in finding solutions.
- Encourage students to use creative approaches.
- Be willing to accept unconventional solutions, more than one solution, or no solution where appropriate.
- Challenge students to think critically, and to justify strategies and solutions.
- Be enthusiastic and recognize the students' desire and perseverance to solve problems.
- Provide appropriate questions and modelling for students.
- Present problem situations that are relevant to other subject areas and everyday life.

Students should not be limited in their problem-solving experiences through deficiencies they may have in reading skills. Ask a student who is particularly adept at drawing to provide a pictorial representation of a problem. Ask another student who has strong verbal skills to reword the problem for other students. While students must be given support in overcoming the reading difficulties they encounter, students should be encouraged to use their stronger learning modalities in developing problem-solving skills.

CLARIFICATION/EXAMPLE

Present problems in ways that will accommodate different learning styles:

- Visual (e.g., provide a picture or diagram)
- Auditory (e.g., read the problem aloud)
- Kinesthetic (e.g., use manipulatives or act out the problem).

Inventories are useful in monitoring student attitudes and beliefs about problem solving. Student reaction to particular statements on an inventory will indicate to the teacher those attitudes/beliefs about problem solving that need further attention and development.

CLARIFICATION/EXAMPLE

Attitude Inventory Items[1]

Mark true or false depending on how each statement describes you in problem-solving situations. There are no right or wrong answers.

_____ 1. I will put down any answer just to finish a problem.
_____ 2. It is no fun to try to solve problems.
_____ 3. I will try almost any problem.
_____ 4. When I do not get the correct answer right away I give up.
_____ 5. I like to try hard problems.
_____ 6. My ideas about how to solve problems are not as good as other students' ideas.
_____ 7. I can only do problems everyone else can do.
_____ 8. I will not stop working on a problem until I get an answer.
_____ 9. I am sure I can solve most problems.
_____ 10. I will work a long time on a problem.
_____ 11. I am better than many students at solving problems.
_____ 12. I need someone to help me work on problems.
_____ 13. I can solve most hard problems.
_____ 14. There are some problems I will just not try.
_____ 15. I do not like to try problems that are hard to understand.
_____ 16. I will keep working on a problem until I get it right.
_____ 17. I like to try to solve problems.
_____ 18. I give up on problems right away.
_____ 19. Most problems are too hard for me to solve.
_____ 20. I am a good problem solver.

Problem-solving attitudes can also be monitored by observing and questioning students as they work in problem-solving situations. Observations can be recorded on individual checklists and rating scales, thus indicating aspects of each student's attitude and performance that require further development. Checklists and rating scales also provide a means for monitoring attitudinal changes and growth through the school year.

[1] Reprinted with permission from *How to Evaluate Progress in Problem Solving*. Copyright 1987 by the National Council of Teachers of Mathematics.

CLARIFICATION/EXAMPLE

Observational Checklist of Problem-Solving Attitudes and Behaviours[1]

_____ 1. Likes to solve problems.
_____ 2. Works cooperatively with others in the group.
_____ 3. Contributes ideas to group problem solving.
_____ 4. Perseveres – sticks with a problem.
_____ 5. Tries to understand what a problem is about.
_____ 6. Can deal with data in solving problems.
_____ 7. Thinks about which strategies might help.
_____ 8. Is flexible – tries different strategies if needed.
_____ 9. Checks solutions.
_____ 10. Can describe or analyze a solution.

CLARIFICATION/EXAMPLE

Observational Rating Scale of Problem-Solving Attitudes and Behaviours[2]

	Frequently	Sometimes	Never
1. Selects appropriate solution strategies.	_____	_____	_____
2. Accurately implements solution strategies.	_____	_____	_____
3. Tries a different solution strategy when stuck (without help from the teacher).	_____	_____	_____
4. Approaches problems in a systematic manner (clarifies the question, identifies needed data, plans, solves, and checks).	_____	_____	_____
5. Shows a willingness to try problems.	_____	_____	_____
6. Demonstrates self-confidence.	_____	_____	_____
7. Perseveres in problem-solving attempts.	_____	_____	_____

[1] Reprinted with permission from *How to Evaluate Progress in Problem Solving*. Copyright 1987 by the National Council of Teachers of Mathematics.

[2] Ibid.

USING STRATEGIES TO SOLVE PROBLEMS

Confidence and ability to solve problems is enhanced when students develop a repertoire of strategies for solving problems. Instruction should build on problem-solving strategies that students already use. As confidence in the use of familiar strategies develops, more sophisticated strategies can be introduced. A sample lesson plan illustrating how specific strategies might be used in the problem-solving process is provided in Resource 1: "Sample Lesson Plan for Problem Solving".

UNDERSTANDING THE PROBLEM

During this stage of the problem-solving process, students must be encouraged to think about the problem before attempting a solution. The teacher can assist students to focus their attention on information and conditions set in the problem by asking appropriate chains of questions. Model and explicitly teach strategies that may be used by students in developing an understanding of the problem situation.

CLARIFICATION/EXAMPLE

Strategies for "Understanding the Problem":

- reading the problem several times
- asking questions
- identifying key words and their meanings
- looking for patterns
- identifying wanted, given, and needed information
- identifying extraneous information
- internalizing the problem by restating in one's own words or by visualizing the problem
- drawing pictures/diagrams
- using concrete manipulatives
- interpreting pictures/charts/graphs
- relating the problem to other problems previously encountered
- simulating or modelling the problem situation.

Teachers can assist students to understand problem situations and to execute appropriate strategies at this stage of problem solving by following the suggestions provided below.

- Have discussions that focus on understanding the problem before students start to work on the problem:
  - What is the question? What do we need to find?
  - What are the conditions/variables in the problem?
  - What data do we need?
- Ask students to explain problems in their own words, or through the use of pictures and diagrams.
- Remind students of similar problems.
- Use coloured markers to highlight important words, phrases or data in the problem.
- Record data provided in the form of a list.

DEVELOPING AND CARRYING OUT A PLAN

In this stage, students should plan strategies for solving the problem, and then use their strategies to actually solve the problem. Explicit teaching of various strategies appropriate to specific problem situations may be necessary. Emphasize that there are often strategies other than computation that can be used to solve problems.

CLARIFICATION/EXAMPLE

Strategies for "Developing and Carrying Out a Plan":

- guessing and checking the result (thus improving the guess)
- using logic or reason
- choosing and sequencing the operations needed
- sorting and classifying information
- applying selected strategies
- presenting ideas clearly
- selecting appropriate calculating/measuring devices and methods
- acting out or simulating the problem
- applying patterns
- estimating the answer
- documenting the process used
- working with care
- working in a group situation where ideas are shared
- visualizing the problem
- speaking to self with positive statements (e.g., 'I can solve this")
- using a simpler problem (making an analogy)
- identifying factors relevant to the problem
- collecting and organizing data into diagrams, number lines, charts, tables, pictures, graphs or models
- experimenting through the use of manipulatives
- breaking the problem down into smaller parts.

The ideas provided below may be useful in helping students who experience difficulty at this stage of the problem-solving process.

- Suggest a solution strategy.
- Give the start of a solution or strategy, and then ask students to complete the solution in order to find the answer.
- Give direct instruction and practice with particular solution strategies.
- Discuss possible solution strategies before students start solving a problem:
  - ask students to suggest reasons why they believe particular strategies might work
  - for one-step and multiple-step problems, ask what action in the problem suggests a particular operation.
- Remind students of similar problems they have encountered in the past.
- Provide a one-step or multiple-step problem without numbers, and ask students to identify the operations used in finding a solution.
- Provide a completed solution to a problem (e.g., a number sentence, organized list, or picture), and ask students to create a problem that would fit the solution.

REVIEWING AND APPLYING RESULTS

This stage encourages students to assess the effectiveness of their solution, and to consider the accuracy of their results. Students should be encouraged to relate answers to the question in the problem in order to verify that the problem has indeed been solved. Evaluation of the strategies used will increase awareness of their appropriateness and of other strategies that might have been used. Encourage students to generalize and apply the strategies they have used to related situations.

CLARIFICATION/EXAMPLE

Strategies for "Reviewing and Applying Results":

- stating an answer to the problem
- restating the problem with the answer
- explaining the answer in oral/written form
- determining if the answer is reasonable
- discussing the process used with others
- suggesting other ways of solving the problem
- checking the answer
- considering the possibility of other answers/solutions
- making and solving similar problems.

The suggestions that follow may assist teachers in planning activities that will develop students' ability to review and assess the effectiveness of their problem-solving efforts.

- Ask students to explain why they chose particular solution strategies.
- Illustrate alternative strategies that might be used to solve the problem, and evaluate their usefulness.
- Discuss incorrect strategies and attempts, and explain why these strategies were inappropriate.
- Use estimation to check the reasonableness of answers obtained.
- Check to see that all relevant information in the problem has been used.

MONITORING AND EVALUATING PROGRESS

Checklists will enable the teacher to determine the extent to which students are using certain strategies in problem solving. By maintaining a class checklist while working on a particular theme, a teacher can diagnose both individual progress in the use of various strategies, as well as total group progress for each strategy emphasized.

CLARIFICATION/EXAMPLE

	Identifies Key Words	Draws Pictures/ Diagrams	Uses Calculator	Applies Patterns	Applies Concepts	Asks Questions	Identifies Relationships	Makes Measurements	Estimates Answer	Explains Process	
Mary		II	IIII	I	I	I	I	III		II	15
Tom	I		I		II		I	III	II	I	11
	1	2	5	1	3	1	2	6	2	3	

In the example above, a tally is inserted in the appropriate column each time the teacher notes tangible evidence of the use of a particular strategy. In this instance, Mary is gaining experience in drawing sketches, making measurements, and in using several other strategies. She has shown no evidence of asking questions or estimating answers. The teacher can plan special lessons to reinforce the strategies that have not been used.

Students need to recognize the importance of reflecting on the strategies they use, on what they have done, and on what they still need to do. Encourage students to monitor and evaluate their own thinking and progress in problem solving through the use of focus questions that ask them to think back and describe how they solved particular problems.

CLARIFICATION/EXAMPLE

Focus Questions For Monitoring and Evaluating Progress[1]

Use the following questions to help you look back and describe your thinking as you worked toward a solution to the problem.

1. What did you do when you first saw the problem? What were your thoughts?
2. Did you use any problem-solving strategies? Which ones? How did they work out? How did you happen to find a solution?
3. Did you try an approach that didn't work and have to stop and try another approach? How did you feel about this?
4. Did you find a solution to the problem? How did you feel about this?
5. Did you check your answer in any way? Did you feel sure it was correct?
6. How did you feel, in general, about this problem-solving experience?

[1] Reprinted with permission from *How to Evaluate Progress in Problem Solving*. Copyright 1987 by the National Council of Teachers of Mathematics.

Students can also be encouraged to reflect upon the thought processes they use by completing a "strategy inventory" that is based on a particular problem-solving experience.

CLARIFICATION/EXAMPLE

Problem-Solving Strategy Inventory[1]

Think about your use of strategies when solving the problem. Check the following that apply.

1. _____ I didn't think about using strategies at all.
2. _____ The idea of using strategies came to my mind, but I didn't think about it much more.
3. _____ I looked at a strategy list, but didn't try a strategy.
4. _____ I looked at a strategy list and picked a strategy, which I tried.
5. _____ I didn't look at a list, but just thought of a strategy to try.
6. _____ I used at least one strategy and it helped me find a solution.

7. I tried the following strategies:

_____ guess and check	_____ solve a simpler problem
_____ make a table	_____ work backward
_____ look for a pattern	_____ draw a picture
_____ make an organized list	_____ write an equation

_____ other _____

Problem solving is often difficult to evaluate because it is process oriented. Be sure to evaluate the problem-solving process that is used, and not just the solution. A possible marking scale has been provided on the page that follows. This scale assigns 0, 1 or 2 points for each stage of problem solving, according to the criteria stated. It should be noted that the marks assigned at any given stage should not be influenced by the marks assigned at other stages of the process.

[1] Reprinted with permission from *How to Evaluate Progress in Problem Solving*. Copyright 1987 by the National Council of Teachers of Mathematics.

CLARIFICATION/EXAMPLE

Marking Scale for Problem Solving[1]	
Understanding the Problem	0: Complete misunderstanding of the problem 1: Part of the problem misunderstood or misinterpreted 2: Complete understanding of the problem
Developing and Carrying Out a Plan	0: No attempt, or totally inappropriate plan 1: Partially correct plan based on part of the problem being interpreted correctly 2: Plan could have led to a correct solution if implemented properly
Reviewing and Applying Results	0: No answer or wrong answer based on an inappropriate plan 1: Copying error; computational error; partial answer for a problem with multiple answers 2: Correct answer and correct label for the answer

Students should not be evaluated on their ability to solve problems simply by what they produce on paper. Attitudes and behaviours such as the following may be taken into account:

- willingness to attempt a problem
- use of a systematic approach
- use of appropriate strategies
- willingness to try other strategies
- logical justification of strategies and solutions
- perseverance in the task
- confidence in ability to solve problems
- willingness to contribute to group problem-solving activities
- willingness to solicit/accept help from others.

Evaluation procedures should include a variety of techniques, including observation, questioning and interviewing. Checklists, inventories and rating scales provide a useful means of recording student attitudes and behaviours as they relate to problem solving. Techniques of evaluation are further discussed in the "Evaluation" section of this manual.

[1] Reprinted with permission from *How to Evaluate Progress in Problem Solving*. Copyright 1987 by the National Council of Teachers of Mathematics.

SOLVING PROBLEMS IN COOPERATIVE LEARNING SITUATIONS

Problem solving provides opportunity for students to develop skills by working in cooperative learning situations. Group work often provides students with a less threatening environment, where they may be more willing to take the risks associated with problem solving. Students participating in a group problem-solving activity can learn new strategies from others, and refine their own problem-solving skills.

Skills often best learned in group settings include the ability to:

- clarify one's own ideas
- evaluate the ideas of others
- compare alternatives.

CLARIFICATION/EXAMPLE

A Paired Problem-Solving Strategy[1]

Students are divided into pairs (problem solver, recorder) in order to work together in solving a problem. The use of a "thinking-aloud" procedure allows the student to see how their partner thinks and solves problems. Thinking steps are thus open to view and can be observed and communicated. The procedure used is as follows:

1. One member (the problem solver) "thinks aloud" while solving a given problem.
2. The other member (the recorder) listens carefully, noting the steps taken in the solving of the problem. At the end of this "think-aloud" procedure, the recorder may ask clarification questions of the problem solver and/or may point out errors made in the problem-solving process.
3. Roles are reversed, repeating the same problem.
4. The modification/extension of both strategies are discussed with each person (or both if agreement is reached) documenting the best "modified" strategy

This strategy could be expanded to include three people by subdividing the recorders' role into recorder and questioner. The three roles would rotate.

Cooperative learning also offers opportunities for enhancing interpersonal skills among students. As students work in group settings, problems in social interaction may arise. A strategy for systematically analyzing a social problem is provided in Resource 2: Social Problem-Solving Strategy. This strategy helps students to identify:

- reasons for the difficulty/conflict
- strategies that may avoid the difficulty/conflict another time.

Teacher modelling and student use of this strategy may improve classroom climate, and enable students to gain confidence in their ability to effectively interact and communicate with one another.

[1] Adapted from SPELT: *Strategies Programme for Effective Learning/Thinking: Inservice Edition*, SPELT International, Ltd., p. 197. Copyright 1987 by R. Mulcahy, K. Marfo, D. Peat and J. Andrews Reprinted by permission.

Teachers can facilitate cooperative problem solving by establishing a "problem-solving corner" in their classrooms. This area of the classroom can be stocked with a variety of interesting problems, puzzles and manipulative materials. Encourage each student to contribute their favourite problem or puzzle to the problem corner. By displaying a "daily puzzle" or "problem of the week", students can be motivated to use the corner on a regular basis as time permits. (Refer to "Using a Math Lab" for additional ideas that may be useful in establishing a problem-solving corner.)

A variety of sample problems are provided in Resource 3: Problem-Solving Ideas. These problems have been selected on the basis of their cognitive demand, as well as effectiveness in developing appropriate strategies for problem solving. While some of the problems are suited only to group settings, many can be solved by students working independently or in small groups. **Teachers are encouraged to develop their own collection of problems/puzzles that are appropriate to curriculum goals and student interest/ability.**

Many worthwhile problem-solving ideas can be found in the following publications:

- *Let Problem Solving be the Focus for the 1980's*, Alberta Education, 1983.
- *Problem Solving Challenge for Mathematics*, Alberta Education, 1985.
- *Problem Solving in Mathematics: Focus for the Future*, Alberta Education, 1987.
- *The Arithmetic Teacher*, National Council of Teachers of Mathematics.
- *The Mathematics Teacher*, National Council of Teachers of Mathematics.

A variety of commercially produced books and kits containing suitable problems and puzzles are also available from local bookstores and libraries.

RESOURCE 1: SAMPLE LESSON PLAN FOR PROBLEM SOLVING

PROBLEM SITUATION

Sue and Ron have cows and chickens on their farm. They have one barn and two chicken coops. Susan told her cousin that they have 14 animals in all. Ron said, "If you count all the legs you get 40". How many of each animal do they have?

I. UNDERSTANDING THE PROBLEM

1. Have a class discussion to understand the problem.

   - Underline key words and important information.
   - Cross out extraneous information.
   - Have a student retell the problem.

2. Ask questions to focus attention on key components of the problem.

   - What animals do they have?
   - How many animals do they have?
   - How many legs does a chicken have?
   - How many legs does a cow have?
   - How many legs did Ron say there were altogether?
   - What do you have to determine?

II. DEVELOPING AND CARRYING OUT A PLAN

1. Decide upon a problem-solving strategy that might be used. Possibilities include:

   - Using a table or chart.
   - Guessing and checking.
   - Drawing a diagram.

2. If students are having difficulty, ask questions that will direct their thought processes to appropriate strategies and considerations.

   - Some students will forget that there are two parameters to the problem (i.e., number of legs and number of animals). "You have the correct number of legs, but how many animals did Sue say they have?"

   - Make an analogy or simpler problem.
     - If there were only six legs, how many chickens and cows are there?
     - After soliciting the correct response, discuss the possibility of zero cows or zero chickens.
     - Extend this analogy to 10 legs and 14 legs until students develop their own plan.

III. REVIEWING AND APPLYING RESULTS

1. Check your answer.

   The solution is six cows and eight chickens. Checking can be done in the following manner:

   $6 \times 4 = 24$
   $8 \times 2 = 16$
   $24 + 16 = 40$ legs

2. Express the answer orally and in written form.

3. Discuss other ways to solve the problem.
   - Organize data into a table.

Cows	Chickens	Legs
14	0	56
13	1	54
12	2	52
11	3	50
10	4	48
9	5	46
8	6	44
7	7	42
6	8	40

The solution is obtained by systematically listing all possibilities until we arrive at the correct number of legs. If there are 14 animals, the number of cows plus the number of chickens must total 14.

   - Apply guesses and checks.

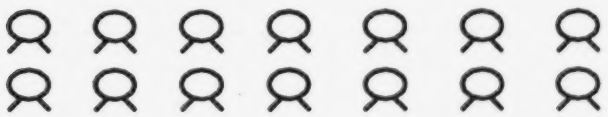
Cows	Chickens	Legs
10	4	48
4	10	36
7	7	42
6	8	40

Using this method, the student can pick any number of cows. If I choose ten cows, this leaves four chickens and a total of 48 legs. This is too many legs, so I choose fewer cows. Repeat this process until you arrive at the correct solution.

- Draw a diagram.

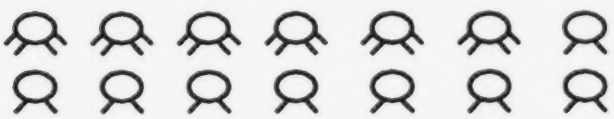
  Step One

  Draw fourteen animals and give them each two legs.



  Step Two

  Add two additional legs to enough animals in order to obtain the required total of 40 legs.



4. Discuss the solution process and apply it to other problems.

   e.g.,
   I have $1.10 in quarters and dimes. Altogether I have eight coins. How many of each coin do I have?

RESOURCE 2: SPOT (SOCIAL PROBLEM-SOLVING STRATEGY)[1]

Teachers are encouraged to use the following social problem-solving strategy as a model when dealing with individual students or as a guide for students to use on their own.

Description of Strategy

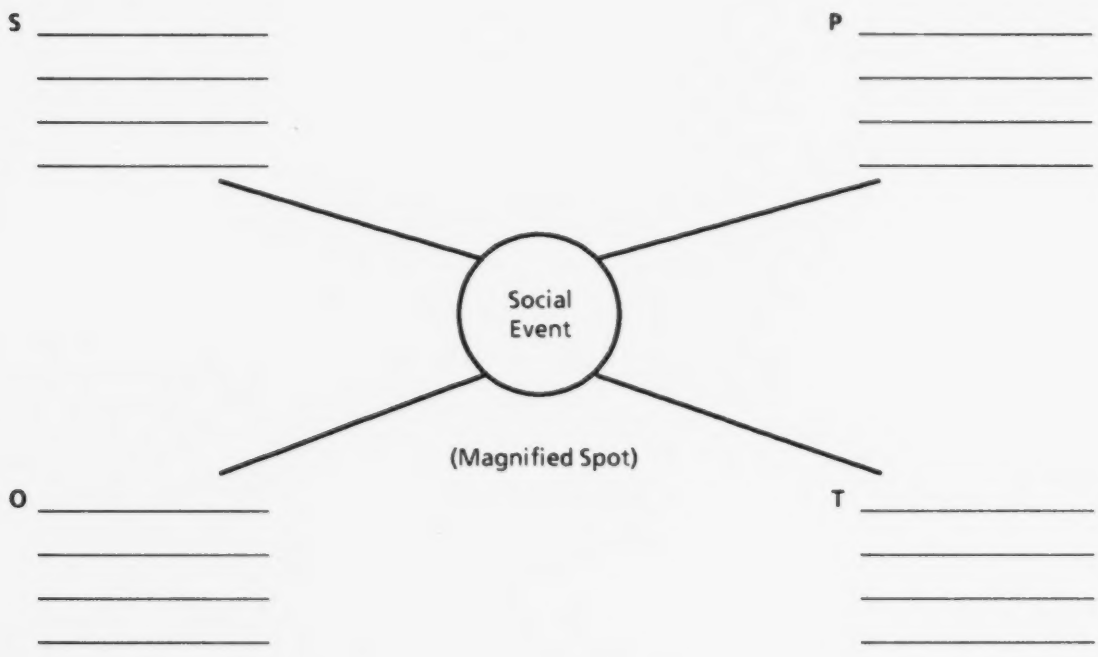
S: Setting: Who? What? Where? When?

P: Problem: What's the situation to be solved?

O: Order of action: What happened?

T: Tail End: What can be done next time?

Problem-Solving Chart:



[1] From *Strategies for Teaching Students with Learning and Behaviour Problems* by Dr. C. Bos, and S. Baughn. Copyright 1988 by Boston, Allyn and Bacon. Reprinted by permission.

RESOURCE 3: PROBLEM-SOLVING IDEAS

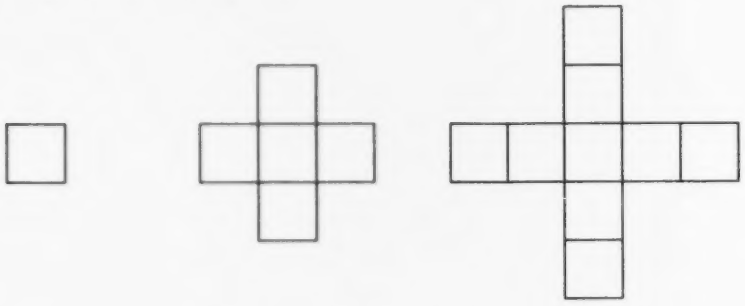
1. LOOK FOR A PATTERN

a. Study the figures below. Look for patterns. Then complete the chart.

Fig. 1 Fig. 2 Fig. 3 Fig. 4

Figure	1	2	3	4	5	6	7	8	9	10
Total Number of Circles	1	3	6	10	?	?	?	?	?	?

b. The 1st figure contains 1 square.
The 2nd figure contains 5 squares.
The 3rd figure contains 9 squares.



1st 2nd 3rd

If you made a drawing of the 4th figure, how many squares would it contain? How many squares would the 10th figure contain?

c. In each case, find the rule that will give the second number if you know the first. Then fill in the rest of the table according to the rule.

8	3
15	10
30	25
50	?
25	?
?	100

5	20
6	24
10	40
8	?
?	100
?	60

2	7
3	10
5	16
8	?
22	?
?	22

d. Complete these patterns:

- 2, 5, 8, 11, _____, _____, _____, _____, _____
- 1, 6, 11, 16, _____, _____, _____, _____, _____
- 64, 32, 16, 8, _____, _____, _____, _____, _____
- 1, 2, 4, 7, _____, _____, _____, _____, _____

2. MAKE A SYSTEMATIC LIST

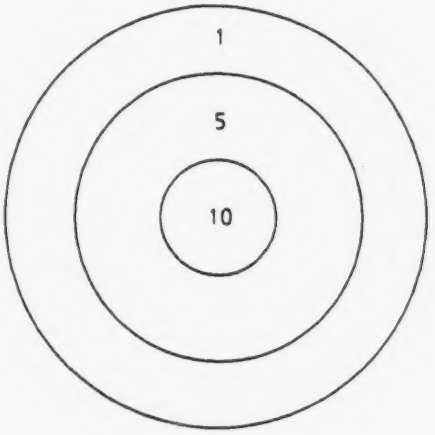
a. Adam, Bill, Carl, and Dean were buying tickets to a movie. In how many different ways could they line up?

Complete this listing:

A B C D
A B D C
A C B D
A C D B
A D B C
•
•
•

b. Suppose you throw three darts and each dart hits the target. There are ten different totals possible. Use a systematic list, like the one below, to help you find the ten possible total scores.

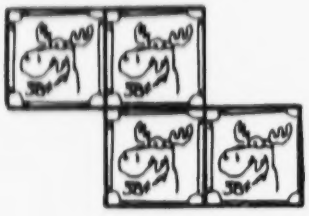
10	5	1	Total
✓✓✓			30
✓✓	✓		25
✓✓		✓	21

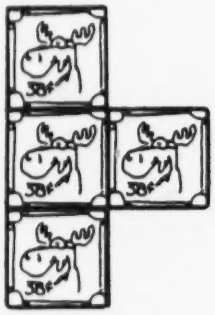


c. There are 6 basketball teams in a tournament. The teams are lettered A through F. Each team plays each of the other teams twice. How many games are played altogether?

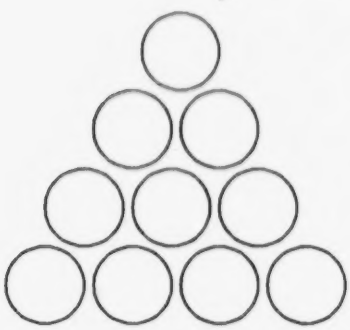
3. USE A MODEL OR DRAWING

a. How many ways can you buy 4 attached stamps at the post office? Make drawings to show at least 10 different ways. Two of them are shown.

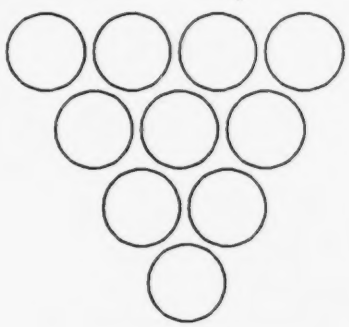




b. Use bottlecaps or markers to make



and move only 3 bottlecaps or markers to make



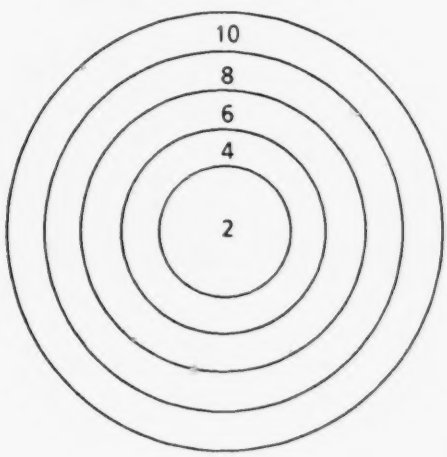
c. This diagram shows 3 views of a special die.
What number is opposite the 5?
What number is opposite the 1?
What number is opposite the 2?

1st View 2nd View 3rd View

4. ELIMINATE POSSIBILITIES

a. Sue threw 5 darts.
Each dart hit the target.
No darts landed on a line.
Which of these scores are you sure is impossible?

38	25	60
30	42	37
26	8	14



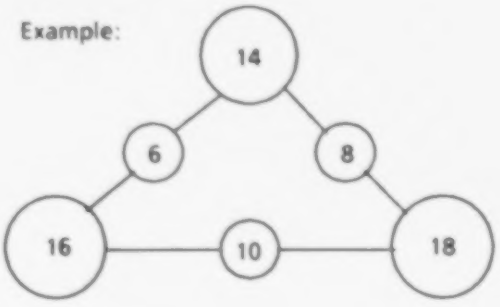
b. Find the ages of my three children.

Clue 1: The product of their three ages is 36.
Clue 2: Two of the children are twins.
Clue 3: The youngest is not a twin.

5. GUESS AND CHECK

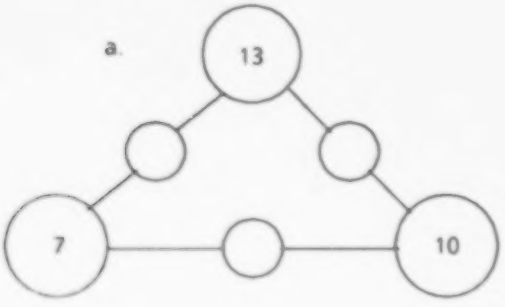
The numbers in the big circles are found by adding the numbers in the small circles.

Example:

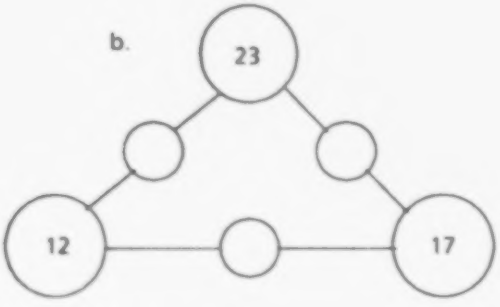


Find the numbers for the small circles in each problem.

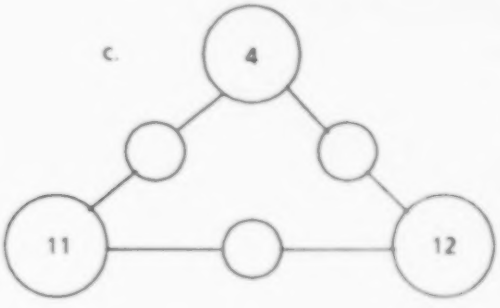
a.



b.



c.



6. A WINNING STRATEGY

Two players, A and B, have a pile of 6 toothpicks. Players alternately remove 1 or 2 toothpicks from the pile. The player who takes the last toothpick wins.

Note to Teacher:
This game does not take long to play so students may be able to play it several times to discern the winning strategy. Students may discover several different strategies for winning. (e.g., The player who makes the second move always does the opposite to what the first player did. That is, if player A makes the first move and removes one matchstick, then player B will remove two and so on.) The teacher may extend this game by changing the original number of toothpicks to 21, and instructing students to move one, two or three toothpicks at a time.

7. WORD PROBLEMS

Place students in pairs. Designate one member as "fact finder" and the other as "problem solver". The "fact finder" will view a simple word problem for one or two minutes and then describe the problem to the "problem solver". The "problem solver" must find a solution without help from the "fact finder". Members of each pair must reverse roles after each problem. The winner in each pair is the person who correctly solves the greatest number of problems.

8. GUESSING AND CHECKING – ELIMINATING POSSIBILITIES

Choose a three- or four-digit number having no digits repeated. Ask someone to guess the number. Provide clues as to how close the guess is to the number chosen:
- say "Zilch" if no digits are correct
- say "Fermi" if there is a correct digit, but in the wrong position
- say "Pico" if there is a correct digit in the correct position.

For example, if the number chosen was 236, and someone guesses 532, the response is pico-fermi (i.e., one digit in the right position and another correct digit in the wrong position).

It may be advisable to choose two-digit numbers when introducing this activity. Students may play the game in pairs, keeping a record of the number of guesses made by each before determining the correct number. The winner is the student who determines the number chosen with the least number of guesses.

9. LOGICAL THINKING

Sue, Ted and Ahmed are seated around a circular table playing a game of Hearts. One of the players is 13, one is 14 and the other is 15 years of age. Each person passes three cards to the right. Ted passes three hearts to the 13 year-old, Sue passes the Queen of spades and two diamonds to the person who passed the cards to the 15 year-old.

How old is Sue?

10. MONEY IN THE BANK

Jane and Arthur each have $10.00 in the bank. Every month Jane plans to add $1.00 to her account. Arthur plans to add $3.00 to his account every month.

(a) In how many months will Arthur have twice as much as Jane?
(b) Solve the problem if they each start with $15.00 rather than $10.00.
(c) Solve the problem if they each start with $20.00.
(d) Describe what you've discovered.

EXTENSION

If they each start with $10.00, when will Arthur have three times as much as Jane?

11. ALGEBRA PROBLEMS

Arthur found this equation in his brother's algebra book:

$15N + 23 = 398$ (Remember, 15N means 15 times N.)

His brother explained that this equation is like a puzzle problem:
I'm thinking of a number. If you multiply it by 15 and add 23 you get 398. What is my number?

CLASS EXERCISES

(a) Arthur's first guess was 12. Was his guess too large or too small?
(b) Arthur's next guess was 30. Was the guess too large or too small?
(c) What is the solution to $15N + 23 = 398$?
(d) Here are other equations Arthur found in the book. First read the equation as a puzzle problem. Then find the solution.

$2N - 4 = 24$

$\frac{4A + 6}{2} = 7$

$\frac{M}{5} + 1 = 3$

$\frac{2N}{5} - 4 = 0$

USE OF TECHNOLOGY

THE CALCULATOR

Technological advances have made the hand-held calculator readily available to all. Students should have ready access to a calculator throughout the mathematics program.

Major benefits to the teaching and learning process that result from regular classroom use of the calculator include:

- calculator use decreases the time spent on tedious computation, thus allowing for an increase in emphasis and time spent on cognitive process and problem solving
- use of the calculator facilitates understanding of number patterns and concepts
- competence in mental arithmetic and estimation will improve through the frequent use of these skills in anticipating and verifying results obtained on the calculator
- the calculator will provide the slower student with the assistance needed to complete certain tasks within the allotted time
- student levels of self-confidence, interest, motivation, and achievement can be expected to increase.

Although most students in junior high school will have had former experience in using a calculator, the teacher should not assume that students are proficient in its use. The calculator, like any other tool, is only effective if used properly. Be prepared to teach students how and when to use the calculator. Calculator activities should place emphasis on:

- understanding of place value/number facts/arithmetical operations
- proper documentation of the numbers and operations used
- estimation and mental arithmetic, and the reasonableness of answers obtained when using the calculator.

A variety of instructional strategies that relate to the topics indicated below have been included in this section of the manual:

- Entry Procedures
- Order of Operations
- Auditory Cueing
- Rounding Results
- Finding Remainders
- Using the Percent Key
- Checking the Reasonableness of Results
- Developing Confidence and Speed

ENTRY PROCEDURES

Provide opportunity for the development of basic entry skills on the calculator. Most errors made by students in using their calculators are due to incorrect entries having been made. Students often enter numbers for division/subtraction in the wrong sequence, and frequently experience difficulty with questions that involve multiple entries.

Ensure that all students are familiar with the basic operations of their calculators. Keys operated include:

[+]	addition	[×]	multiplication
[−]	subtraction	[C]	clear
[÷]	division	[=]	equals.

Students may require practice in entering numbers on their calculators, and also in performing arithmetical operations. Demonstrate appropriate procedures through the use of an overhead projector/transparency. Provide opportunity for students to develop confidence in using these procedures through the use of both oral and written exercises.

CLARIFICATION/EXAMPLE

Whole Number Entry With Floating Decimal
(e.g., 375)

Entry		Display
Turn on		0.
Enter	3	3.
Enter	7	3 7.
Enter	5	3 7 5.

Decimal Entry
(e.g., 0.45)

Entry		Display
Turn on		0.
Enter Decimal Point	.	0.
Enter	4	0. 4
Enter	5	0. 4 5

CLARIFICATION/EXAMPLE

Procedure For Arithmetical Operations
(e.g., 40 + 30)

	Entry	Display
Enter	4 0	40.
Press	+	40.
Enter	3 0	30.
Press	=	70.

ORDER OF OPERATIONS

Order of operations must be emphasized when working with multiple step calculations and problems. Make a poster that illustrates order of operations, and provide students with a mnemonic device that will facilitate recall of the skills they develop.

CLARIFICATION/EXAMPLE

B	O	D	M	A	S
↓	↓	↓	↓	↓	↓
Brackets	of	Divide	Multiply	Add	Subtract

The importance of the order of operations can be reinforced through the applications that follow. Although each problem may be solved through one set of key strokes on some calculators, students should be encouraged to document the process used and record intermediate solutions.

CLARIFICATION/EXAMPLE

1. Jane used her calculator to compute the number of calories in the snack she ate after school. She had a coke with 145 calories, 18 pretzel sticks with 4 calories each, and 7 potato chips with 12 calories each. She entered the following numbers on her calculator:

   145 + 18 x 4 + 7 x 12 =

   To her horror, the calculator showed 7908 calories! Jane knew that her allowance of calories for an entire day was only 2280 calories. What went wrong?

2. What is the cost of the following restaurant bill?

   2 cokes @ $0.85 =
   3 orders of fries @ $0.99 =
   1 hamburger @ $2.59 =

   TOTAL: _____

Activities that may further develop an understanding of appropriate calculator procedures for performing sequential operations are provided in Resource 1: Using Brackets and Parentheses.

AUDITORY CUEING

The wording of subtraction and division operations may often cause confusion. Variations in wording may cause students to enter numbers into the calculator in the wrong sequence. Be consistent with the wording/auditory cues you provide for students. Students should develop the ability to handle alternative auditory cues, but only after basic skills and procedures are understood.

CLARIFICATION/EXAMPLE

Alternative Auditory Cues for Subtraction

- subtract 3 from 6
- 6 minus 3
- take away 3 from 6
- 6 take away 3
- What is the difference between 6 and 3, or 3 and 6?

Alternative Auditory Cues for Division

- divide 6 by 3
- 6 divided by 3
- 3 divided into 6
- What is 6 over 3?

ROUNDING RESULTS

Students need to learn how to interpret the results of computations performed on the calculator. This is particularly true in division, where digits are displayed to the right of the decimal points when the answer is not a whole number. Encourage students to consider the context in which the computation was performed, and to determine appropriate procedures for rounding (i.e., should the answer be rounded to the correct tenth or hundredth?)

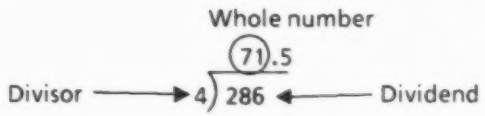
The "Computational Facility and Estimation" section of this manual provides strategies that may be useful in developing students' ability to round numbers.

FINDING REMAINDERS

Division may be a difficult operation when performed by hand or done on the calculator. If the divisor is a factor of the dividend, the process is relatively simple when performed on the calculator. If it is not, however, students must recognize whether the answer is to be expressed as a decimal (which may require rounding), or as a whole number with remainder. As there are some problem situations where exact remainders need to be known, students should be familiar with procedures for finding whole number remainders when performing division on the calculator.

CLARIFICATION/EXAMPLE

What is the whole number remainder for 286 ÷ 4?



Step 1: Multiply the divisor by the whole number in the answer.

$4 \times 71 = 284$

Step 2: Subtract the result of step 1 from the dividend to obtain the remainder.

$286 - 284 = 2$ (remainder)

Students may benefit by initially documenting the process they use for finding whole number remainders in a table or chart. The table provides a structure for the process, and enables students to record intermediate steps and solutions in an effective way.

CLARIFICATION/EXAMPLE

Question	Calculate Answer	Step 1 Multiply Divisor by Whole Number	Step 2 Subtract Answer in Step 1 from Dividend	Remainder
$8\overline{)47}$				
$4\overline{)550}$				
$2\overline{)961}$				
$62\overline{)4217}$				

USING THE PERCENT KEY

Most hand-held calculators have a percent key. Although this key is very useful, the percent function does not operate in the same way on all calculators.

There are two main types of calculators, each using a different logic system as its base. One is "algebra-based" and the other is "arithmetic-based". The calculators do not differ in appearance, but in how calculations are executed. While this duality of logic systems does not present a problem in most instances, it does affect how the percent function operates:

- if the calculator is algebra-based, pressing the percent key causes the number in the display to be divided by 100
- if the calculator is arithmetic-based, the percent key is only used as a multiplier.

Encourage students to determine which logic system their calculator uses by doing the following:

- enter the number 25, and press the percent key
- if the display reads 0.25, the calculator is algebra-based
- if the display does not change, reads 0 or E, the calculator is arithmetic-based.

CLARIFICATION/EXAMPLE

Find 25% of 36.

Keystrokes on algebra-based calculator:

[25] [%] [×] [36] or [25] [×] [36] [%]

Keystrokes on arithmetic-based calculator:

[36] [×] [25] [%]

CHECKING THE REASONABLENESS OF RESULTS

Encourage students to always anticipate and verify the results they obtain on their calculators:

- Is my answer reasonable?
- Within what range of numbers must my answer lie?

Model strategies that will enable students to estimate the results of computations they are about to perform on the calculator (see "Computational Facility and Estimation").

CLARIFICATION/EXAMPLE

Question	Estimated Answer	Calculated Answer
48 + 50 = $10.00 – $1.96 = 52 × 9 = 37 ÷ 7 = 26 × 9 = $32.19 – $3.82 = 47 + 80 = $3.01 ÷ $0.98 =		

CALCULATOR GAME OF 'ESTIMATION' (2 players)

Two players each have 30 seconds to estimate a product or quotient of two, three or four digit numbers. The player with the closer estimate wins a point. The first player with ten points wins.

DEVELOPING CONFIDENCE AND SPEED

Students may initially feel more confident in performing computations with paper and pencil. Confidence and skill in performing computational procedures on the calculator will increase as students gain experience in using the calculator in a variety of contexts:

- drill and practice
- timed challenges
- problem-solving activities
- puzzles and games.

CLARIFICATION/EXAMPLE

CALCULATOR GAME OF '21' (2 players)

On each turn a player can push only one of the three keys [1], [2] or [3] along with the [+] key. The first player to reach 21 is the winner. Ask students to determine the winning strategy.

CALCULATOR GAME OF '50' (2 players)

This game is similar to the game of '21'. Players may use the [1], [2], [3], [4], [5] and [6] keys. The first player to reach 50 is the winner.

Additional puzzles and games that may be useful in developing student confidence and ability to use the calculator have been provided in Resource 2: Calculator Puzzles and Games).

THE COMPUTER

Literacy, in our present age of technology, has taken on a new meaning. No longer is the ability to read and write sufficient for many people. There is an ever-increasing need to be computer literate.

Classroom use of the computer is increasing. According to the National Council of Teachers of Mathematics, every mathematics classroom should either have a computer or have ready access to a computer.

The computer can be motivational, whether used in demonstration or actual instruction. Its graphic capabilities can help to bring mathematics alive for students. In addition to its motivational value, the computer may:

- contribute to the development of concepts and skills
- provide drill and practice
- eliminate some of the tedium involved in many mathematical situations
- generate large amounts of data in short periods of time
- contribute to the development of problem-solving strategies and skills.

There are several "user-friendly" programming languages currently available for use. These programs enable both teacher and student to use the computer without relying solely upon commercially prepared software that may be expensive. Two of these programming languages are briefly discussed on the pages that follow:

- BASIC
- LOGO.

BASIC has been selected because most microcomputers utilize this language. LOGO has been selected because of its simplicity and graphic capabilities. Although neither BASIC nor LOGO require extensive programming knowledge, the degree to which these programs are used within the mathematics program will depend upon local needs and resources.

COMPUTER LANGUAGES: BASIC AND LOGO

Simple programs in BASIC and LOGO are referenced throughout this manual. This section provides the reader with the commands and procedures needed to run these programs. Teachers may wish to modify the BASIC and LOGO programs that have been provided to better meet students' needs, interests, and abilities. Teachers need only minimal knowledge of the computer language in order to use the programs and make desired changes.

The use of BASIC may vary slightly in response to the type of computer being used. The programs described throughout this manual and the information provided on the following pages all refer to the BASIC programming language used on an Apple computer. Different versions of LOGO are also available. Although the versions are very much alike, there are some small differences. The version described here is Terrapin LOGO.

BASIC COMPUTER LANGUAGE

Most microcomputers are capable of interpreting BASIC language commands. A disc is not needed to boot the computer for BASIC as the computer will automatically interpret the commands. A disc is only needed if the program is to be saved. When using the computer without a disc in the drive, the drive will "whir" for a time before the cursor appears. To stop the whirring, press "control reset". The cursor should appear in the lower left hand corner.

A list of common commands used in BASIC programming is provided below.

Key Word	Description
DATA	DATA 45, 67, 35 Holds the data for use by READ statement.
END	Terminates the program.
FOR	FOR X = 1 to 10 Substitutes the numbers from 1 to 10 into the program. Causes a LOOP to occur.
GOTO	GOTO 50 Tells the computer to go to line 50 of the program.
HOME	Clears the screen and sends the cursor to the upper left hand side of the screen.
IF . . . THEN	IF X > 1 THEN 90 The computer makes a decision as to the truth of a statement and if true, in this example, will proceed to line 90. If the condition is not true it will go to the next line.

Key Word	Description
INPUT	INPUT :N$ Calls for input from the keyboard during program execution.
INT	PRINT INT (23.6) Will give the largest integer less than or equal to the value given. In this case the computer would print 23.
LET	LET X = 10 Assigns a value to a variable.
NEXT	NEXT X Closes a FOR . . . NEXT loop.
PRINT	PRINT "HELLO" The computer will print the expression in the quotation marks.
READ	READ X, Y, Z Reads values from a DATA statement.
REM	REM - - "THIS IS A COMPUTER PROGRAM" The REM command allows the programmer to insert remarks for explanation into the program.
STEP	FOR X = 0 to 100 STEP 10 The computer will substitute the numbers from 0 to 100, counting in increments of 10.

The arithmetic commands in BASIC include:

+	– addition
-	– subtraction
*	– multiplication
/	– division
^	– exponentiation

Additional commands that may be needed to program in BASIC include:

Command	Execution
LIST	Displays the entire program contained in the computer's memory. If the program is long it may be necessary to only display part of it at a time. This can be accomplished by listing the lines to be displayed. For example, LIST 10-100 would display all lines from 10 to 100.

Command	Execution
LOAD	To retrieve a program from a disc, type LOAD and the name of the program.
NEW	Before entering a program, type NEW to clear the computer's memory.
RUN	After a program has been entered type RUN to have the computer execute the program.
SAVE	If the program is to be saved on a disc, type SAVE and the program name.

When programming in BASIC, the computer executes the commands in order of their line numbers. It is important that each line be numbered and that they are numbered sequentially with the order of the commands.

CLARIFICATION/EXAMPLE

A BASIC program for finding the factors of a number:

```
 10  REM FIND ALL FACTORS OF A NUMBER
 20  PRINT "WHAT IS THE NUMBER"
 30  INPUT N
 40  PRINT "THE FACTORS ARE:"
 50  FOR K = 1 TO N
 60  LET X = N/K
 70  LET Y = INT (N/K)
 80  IF X = Y THEN PRINT K
 90  NEXT K
100  END
```

Notice that the line numbers are listed in increments of ten. This allows the programmer to insert a new line without having to re-type the rest of the program. For example, if the programmer wished to insert a new line between 10 and 20 it could be entered as line 15 and the computer will automatically execute the new line in its numerical order.

Here is an explanation of the program.

Line Number	Execution
10	The REM command has allowed the programmer to state the purpose of the program. Note that the REM statement has no effect on the running of the program. It is only there for explanatory purposes.
20	When the program is run the statement "WHAT IS THE NUMBER" will be printed.
30	INPUT N calls for the operator to input the number to be factored.

CLARIFICATION/EXAMPLE

Line Number (continued)	Execution (continued)
40	After the operator inputs the number, the computer will print the statement: "THE FACTORS ARE". Note that the statement to be printed is enclosed in quotation marks.
50	FOR K = 1 to N tells the computer to test the numbers from 1 to N (the number specified in INPUT) in increments of one. Notice that the STEP statement does not need to be included if the step is one.
60	This LET statement defines the variable X as the quotient of N ÷ K.
70	This LET statement defines the variable Y as the integer value of N ÷ K. The integer value is the greatest integer not larger than N ÷ K.
80	The conditional statement IF . . . THEN tests a condition and begins a loop. In this case the computer is determining if X = Y. That is, when N is divided by K, does it result in an integer (no remainder)? If the statement is true then K will be printed.
90	NEXT K tells the computer to test the next value of K. Remember, the computer will start testing at 1 and end at N.
100	When the computer reaches the END command it will cease executing the program.

Run the previous program to be sure that it works. A sample of a run would appear as:

Type RUN

The screen should read:

WHAT IS THE NUMBER

Enter a number (e.g., 36). The computer screen will now look like this:

THE FACTORS ARE:
1
2
3
4
6
9
12
18
36

Once a program has been run, the teacher may wish to make revisions to the program. These revisions often involve adding additional lines and commands to the original program.

CLARIFICATION/EXAMPLE

A revised BASIC program for finding the factors of a number:

```
 10  REM FIND ALL FACTORS OF A NUMBER
 20  PRINT "WHAT IS THE NUMBER"
 30  INPUT N
 40  PRINT "THE FACTORS ARE:"
 50  FOR K = 1 TO N
 60  LET X = N/K
 70  LET Y = INT (N/K)
 80  IF X = Y THEN PRINT K
 90  NEXT K
100  END

 25  PRINT "YOU WISH TO FACTOR?"
 95  PRINT "DO YOU HAVE ANOTHER NUMBER?"
 96  INPUT R$
 97  IF R$ = "YES" THEN 20
 98  PRINT "OK!  GOOD BYE FOR NOW."
```

Line 25 will be inserted between lines 20 and 30. It will cause the computer to print YOU WISH TO FACTOR? This insertion only serves to clarify the statement WHAT IS THE NUMBER. The entire statement could not be placed on line 20 because there are only 38 character spaces on a line.

Line 95 is the first of three lines that allows the program to be repeated. When the program is run after the initial list of factors, the computer will ask if another number is wanted.

Line 96 calls for input from the operator. Note the form of the input, R$. This is called a string variable. It prepares the computer for input which may be alphabetic or numeric, and which may be several characters in length. If the input is to be a word or letter, a string variable must be used.

Line 97 is a conditional statement. If the input from the operator is YES (not just Y), the computer will go back to line 20 and begin the factoring process again. If the input is anything other than YES the computer will go on to the next line.

Line 98 causes the computer to print "OK! GOOD BYE FOR NOW.".

The examples and explanations included in this section are fairly simple and include only those commands that a computer novice would use. Teachers may wish to consult other programming texts for additional commands and strategies that are used when programming in BASIC.

LOGO COMPUTER LANGUAGE

The LOGO computer language is an interpretive language in that commands may be executed immediately, or can be stored for use in "procedures". A procedure is a group of one or more instructions and commands that the computer stores for future use. Once a procedure has been defined, it may be used as a part of other procedures.

There are many facets to the LOGO language, but perhaps the most appealing characteristic of LOGO is its graphic capability. Very little knowledge of the language is required prior to using the graphics mode of LOGO. This section will highlight some of the graphics commands to be used in Terrapin LOGO. Be prepared to make adjustments to these commands if using another version of LOGO (e.g., LOGO Writer).

To enter the graphics mode, type DRAW. The LOGO turtle, Δ, should now appear in the middle of the screen. This is the turtle's HOME position. Once you are in the draw mode you can command the turtle to perform various moves. Some of the more commonly used graphics commands are provided below.

Graphics Command	Execution
HOME	This command places the turtle in his home position in the center of the screen.
FD _____*	This command followed by a number will move the turtle forward that number of turtle steps. For example, FD 100 would move the turtle forward 100 steps.
BK _____*	This command followed by a number will move the turtle backwards by that number of turtle steps.
LT _____*	This command followed by a number will designate a left turn by that number of degrees. For example, LT 45 will turn the turtle to the left 45 degrees.
RT _____*	This command will turn the turtle by the desired number of degrees to the right.
PU	Means "pen-up", and ensures that the turtle's path that is executed after the PU command will not show.
PD	Means "pen-down". If a PU command has been used, it is necessary to type PD prior to any further commands to make the turtle's path be visible.
CS	Means clear screen. This command erases all previous paths from the screen.

* Do not type the blanks. The blank lines represent a need for numerical input.

To access the text mode of LOGO for programming, simply type TO and the name of your procedure. LOGO will automatically switch into the text mode.

The edit commands listed below will be useful in defining procedures and writing simple programs.

Edit Command	Result
CONTROL D (CTRL D)	Deletes character under the cursor.
CTRL P	Moves the cursor up one line.
CTRL N	Moves the cursor down one line.
CTRL E	Moves cursor to the end of the line.
CTRL G	Stops the procedure .
CTRL C	Defines the procedure.

CLARIFICATION/EXAMPLE

A sample procedure to be executed in LOGO:

```
TO   SQUARE :S
FD   :S
RT   90
FD   :S
RT   90
FD   :S
RT   90
FD   :S
RT   90
END
```

Explanation:

TO SQUARE :S

The TO tells the computer that you are going to define a procedure which you are naming SQUARE. Procedure names can only be one word. If it is necessary to have more than a one word description, the words may be separated with periods (e.g., TO SQUARE.MAZE). The :S indicates to the computer that there will be a variable in the procedure. In this case the variable is the length of the side.

The directions that follow instruct the turtle to move forward a specified number of steps, followed each time by a 90° right turn. Note that the final RT 90 is not necessary for drawing the square, but it is good programming practice to have the turtle end in the home position facing forward.

To run the procedure simply type SQUARE and a length of a side (e.g., SQUARE 10). The computer should now draw a square whose sides are 10 turtle steps in length.

The procedure for square could also be written as:

```
TO SQUARE :S
REPEAT 4 [FD :S  RT 90]
END
```

Defining the procedure this way makes use of the REPEAT command and shortens the program.

Once a procedure has been defined in LOGO, it may be used in other procedures.

CLARIFICATION/EXAMPLE

```
TO STACK.SQUARE :N :S
REPEAT :N [SQUARE :S FD :S]
END
```

If STACK.SQUARE 5 20 is entered, the computer will draw five squares whose side length is 20 turtle steps.

After entering a procedure, be sure to press CTRL C to have the computer put the definition into its memory. If you wish to save a procedure on a disc, type SAVE "FILE NAME". To read from the disc, type READ"FILE NAME".

This brief description of LOGO is intended to enable teachers and students to begin using the graphics and text modes of Terrapin LOGO. Only the graphic capabilities of LOGO have been discussed. The arithmetic capabilities of the language could be investigated through the use of a LOGO manual.

RESOURCE 1: USING BRACKETS AND PARENTHESES

A. Calculate 165 + (168 × 337)

First multiply: (168 × 137) = _____

Then add: 165 + 56 616 = _____

Here is the sentence to use. 168 × 337 + 165 = _____

B. Rewrite the following in proper calculator sequence. Then do the calculation.

	Calculator Sentence	Answer
1. 2411 + (3768 × 72)	_____	_____
2. 180 164 × (165 + 28)	_____	_____
3. 273 × (8927 − 7373)	_____	_____

If there is more than one set of parentheses, sometimes square brackets are used.

[41 + (43 × 44)] ÷ 40
↑ ↑
Next Do first

Do everything in round brackets first, followed by everything in square brackets. Then do the rest of the work.

C. Complete the following:

	Calculator Sentence	Answer
1. [36 × (71 + 32)] − 5 =	_____	_____
2. [57 + (472 − 150)] ÷ 8 =	_____	_____
3. 586 + [35 + (483 ÷ 7)] =	_____	_____

RESOURCE 2: CALCULATOR GAMES AND PUZZLES

A. Your calculator can spell for you. Certain numbers on your calculator, when turned upside down, look like letters.

1. Find out which numbers produce the following letters:

   O___ I___ E___ S___ L___ B___ G___ Z___

2. What word does each of the following numbers spell?

   77 345 _____ 0.7734 _____

   3504 _____ 3080 _____

3. Solve each of the following number problems to find the word suggested by the clue.

   - a telephone greeting: (1 – .93394) ÷ .9 = _____ (hello)
   - what a snake does: 471 × 265 + 410 699 = _____ (hisses)
   - what water does at 100°C: 305 644 ÷ 43 = _____ (boil)
   - where a turtle lives: 11 345 = _____ (shell)
   - the name of a book: 4202 × .5 × 18 = _____ (bible)
   - how math is for you: 16 × 3 × 5 × 83 × 277 = _____ (oh bliss)

4. Solve these expressions in order to answer the questions:

   - What did Amelia Earhart's father say the first time he saw her fly an airplane?

     (.023 × 3 + 10 141) × 5 = _____ (she solos)

   - How do fish breathe?

     5787 ÷ 3 × 4 = _____ (gill)

   - What is a pit?

     59 × 59 + 223 = _____ (hole)

5. Create other words and phrases that are the results of pre-planned calculations.

B. Guess the Number.

1. Ask a friend to enter any number (not more than six digits) on the calculator and then multiply by 2, add 4, multiply by 5, add 12, multiply by 10 and press the " = " key.

2. Take the calculator back from your friend and subtract 320. The result will end in one or more zeros. Drop the zeros, and you'll have the number your friend started with.

Example: Choose 65219

- 65 219 × 2 + 4 × 5 + 12 × 10 = 6 522 220.
- 6 522 220 − 320 = 6 521 900; discard the two trailing zeros, and you have 65 219, the number your friend started with.

C. Change in Your Pocket.

1. Ask someone to take all the change out of their pocket and count it (in cents). Ask the person to enter the amount into a calculator, multiply it by 10, add 1, multiply by 2, add 21 and press the " = " key.

2. Take the calculator back and multiply by 5. The result will be a number ending in 15. Discard the 15 and subtract 1 from what is left. The answer will be the original amount of change in the person's pocket.

Example: 43 cents change

- 43 × 10 + 1 × 2 + 21 = 883.
- Take 883 × 5 = 4415; discard the 15 to get 44; subtract 1 to get 43 cents. Your friend had 43 cents in his or her pocket.

D. Your Birthday and Age.

1. Hand someone your calculator and ask them to enter the number representing the month of their birthday; multiply it by 100, add their date of birth, multiply by 2, add 9, multiply by 5, add 8, multiply by 10, subtract 422, add their age and then press the " = " key.

2. Take the calculator back and subtract 108. The result will be a five- or six-digit number. The first digit(s) tell the month of birth; the next two the date of birth; and the last two, the age of your friend. Your friend will be impressed.

Example: Your friend is 15 years old and was born on March 13.

- 3 × 100 + 13 × 2 + 9 × 5 + 8 × 10 − 422 + 15 = 31 423.
- Take 31 423 − 108 = 31 315 or 3/13/15.

Your friend was born in the third month (March), on the 13th day, and is 15 years old.

COMPUTATIONAL FACILITY AND ESTIMATION

Technology has caused the emphasis in the skills required for computational competence to change over the last decade. Today, computational facility includes more than the knowledge and skills required to perform paper-and-pencil computations with standard algorithms. While these skills are important, responsible participation at home and at work also requires facility in performing mental arithmetic, in the application of calculator skills, and in applying strategies of estimation. Surveys show that mental computation and estimation are used in more than 80 percent of all real life problem–solving situations outside the classroom.

Teaching and learning must focus on the development of a variety of computational strategies that include:

- paper-and-pencil process. Paper-and-pencil computation should emphasize the understanding of process, and de-emphasize calculation with large numbers. Addition and subtraction should generally include numbers with no more than three digits. Multiplication and division should be performed on numbers containing up to three digits, using multipliers/divisors of no more than two digits
- the use of mental arithmetic. Related activities should encourage knowledge and recall of basic facts, as well as motivate students in the application of these facts to more sophisticated processes. Short drill and practice activities that become part of the daily routine should promote the development of mental arithmetic skills and foster a habit for their use
- the use of a hand-held calculator. Calculator activities should emphasize an understanding of place value and the ability to judge the reasonableness of the results of calculations. Skill in estimation and mental arithmetic are important in enabling the student to anticipate and verify calculator results
- the use of estimation. Skill in estimation will enable students to be alert to the reasonableness of computational results, and determine whether particular results are precise enough for the purpose at hand. In order to carry out rapid estimations, students must understand place value, have skill in single-digit operations, be able to multiply and divide by powers of ten, and have facility in rounding to the number of significant digits required by the situation.

The instructional strategies provided in this section of the manual that may be used in developing student ability to compute by a variety of methods include:

- Developing Computational Process
  - Understanding Place Value
  - Using Manipulatives
- Developing Mental Arithmetic Skills
  - Recalling and Applying Basic Facts
  - Using Formal Strategies
- Developing Strategies for Estimation
  - Front-End Estimation
  - Rounding
  - Compatible Numbers
  - Clustering (or Averaging).

Strategies useful in developing calculator skills are provided in the "Use of Technology" section of this manual.

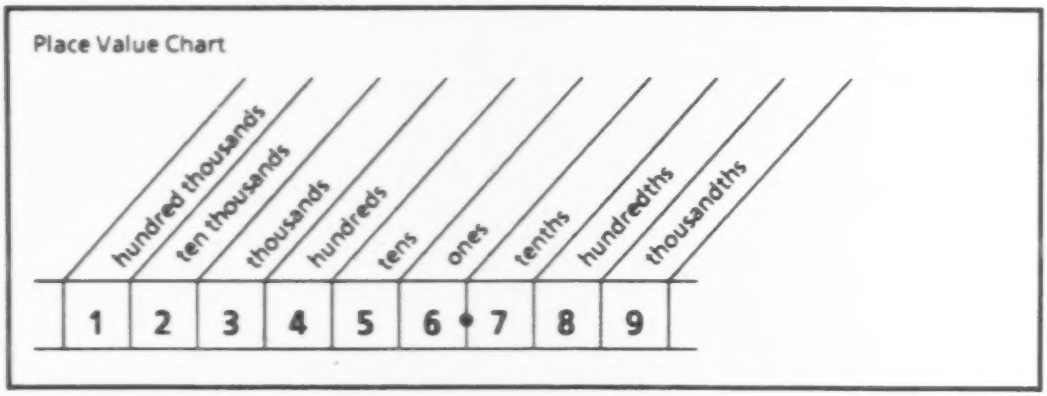
DEVELOPING COMPUTATIONAL PROCESS

UNDERSTANDING PLACE VALUE

An understanding of place value is critical to the development of computational process. Develop an understanding of whole number and decimal place value through discussion, questioning and concrete/visual experience.

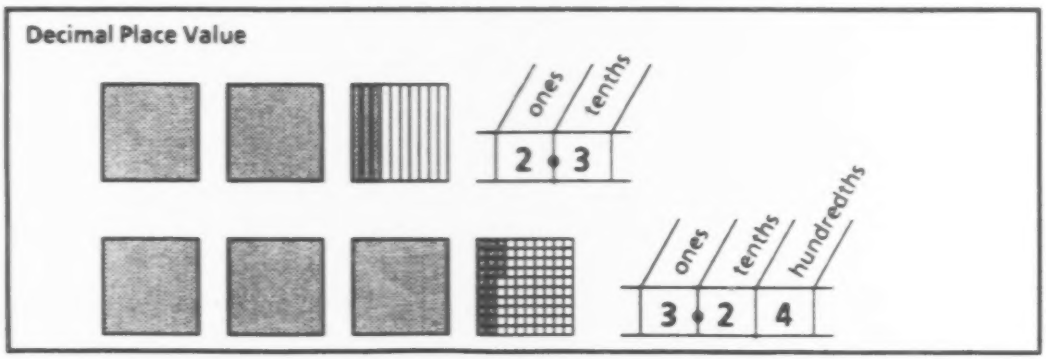
- Teachers may find it beneficial to have a large place value chart on display in the classroom. Ask students to construct their own place value charts for personal reference throughout the program.

CLARIFICATION /EXAMPLE



- Use visual and manipulative materials as required to develop/reinforce student understanding of whole number and decimal place value:
  - base ten blocks
  - grid paper.

CLARIFICATION/EXAMPLE



- Design activities that require students to apply their knowledge of place value to computations performed on the calculator.

CLARIFICATION/EXAMPLE

- Enter the number "21 455" into a calculator.
- Suggest an amount that can be added/subtracted to this number in order to obtain the following results:

  26 455 (+ 5 000)
  21 405 (– 50)
  61 455 (+ 40 000)
  21 055 (– 400).

- Verify each idea by performing the necessary operations on the calculator.

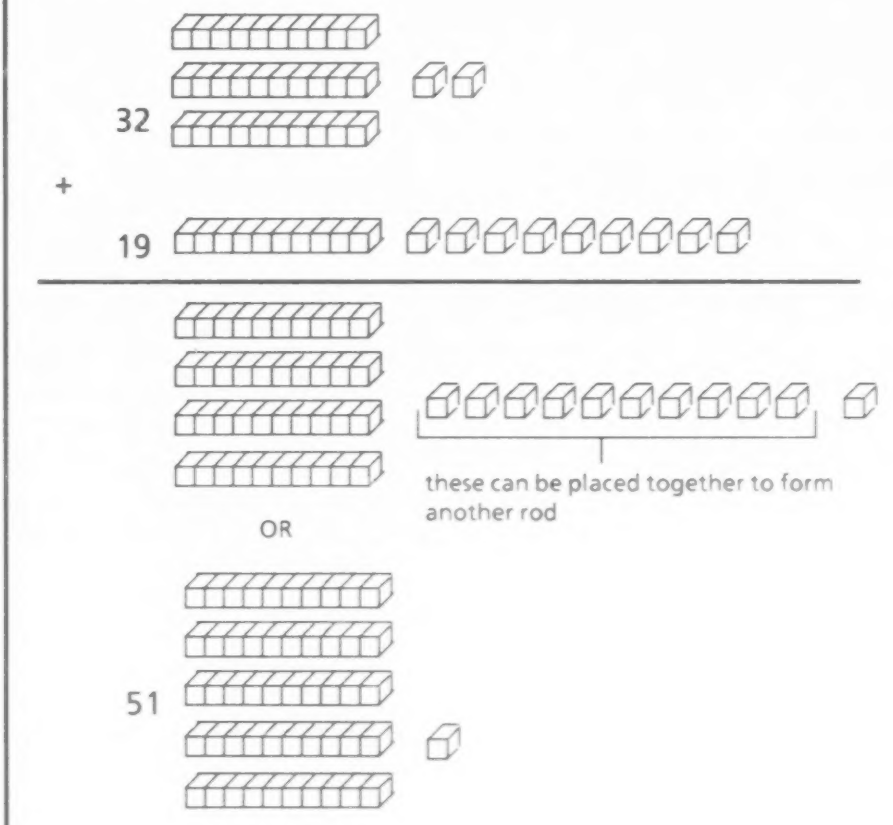
USING MANIPULATIVES

Manipulative materials will provide students with a concrete base upon which to build concepts and skills. Students experiencing difficulty with arithmetical process should be given opportunities to use manipulatives in developing an understanding of number operations and relationships.

The examples which follow illustrate how base-ten blocks might be used to reinforce students' understanding of whole number and decimal operations. In many instances, teachers can improvise and use other readily available materials (e.g., blackline masters, strips of coloured paper, coloured chips) to develop an understanding of related processes. Once processes are understood, students should become proficient in their application through paper-and-pencil algorithms involving numbers of limited size.

CLARIFICATION/EXAMPLE

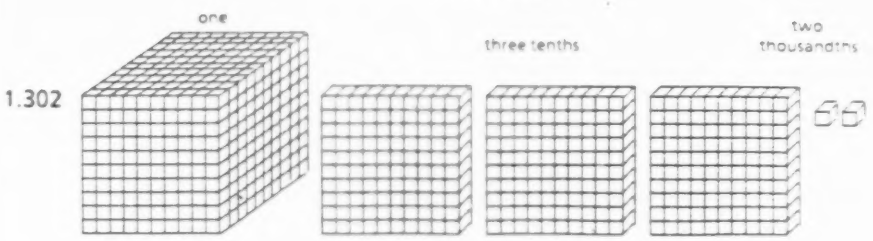
Addition of Whole Numbers



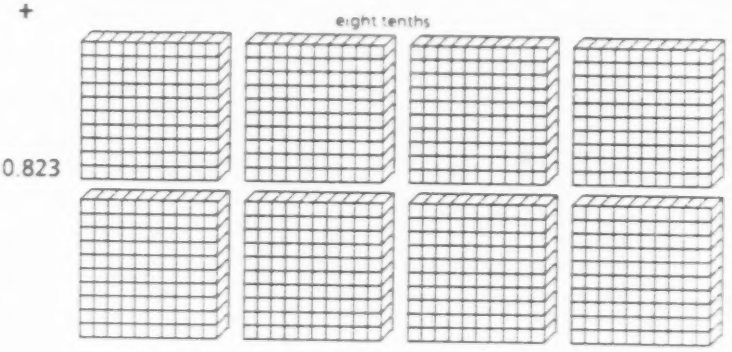
CLARIFICATION/EXAMPLE

Addition of Decimals

1.302 + 0.823

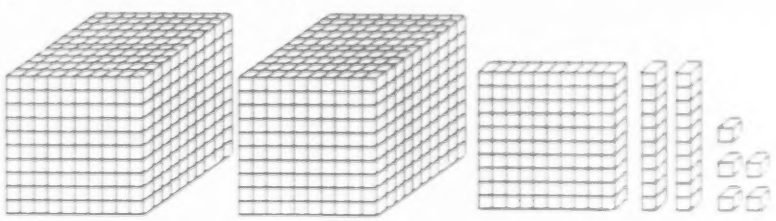


+



2.125

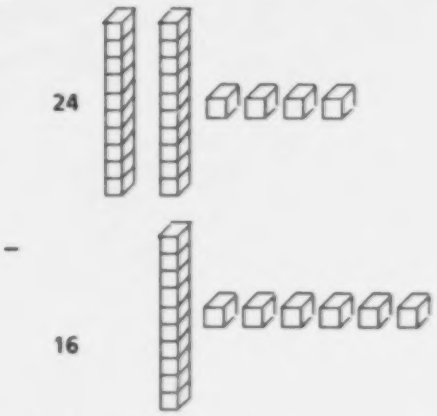
By stacking the "tenth squares" together, we form:



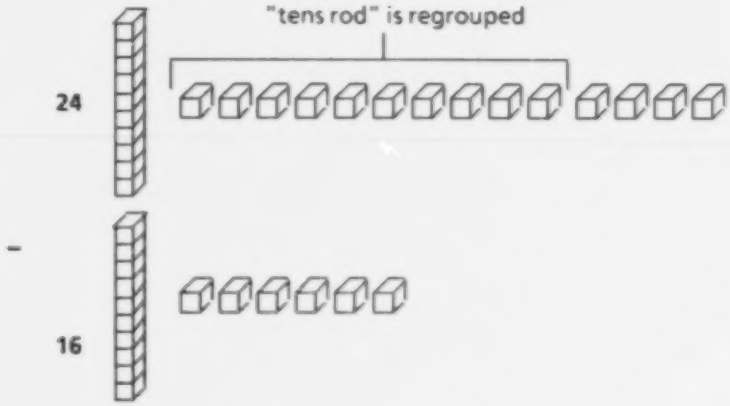
The answer is 2.125

CLARIFICATION/EXAMPLE

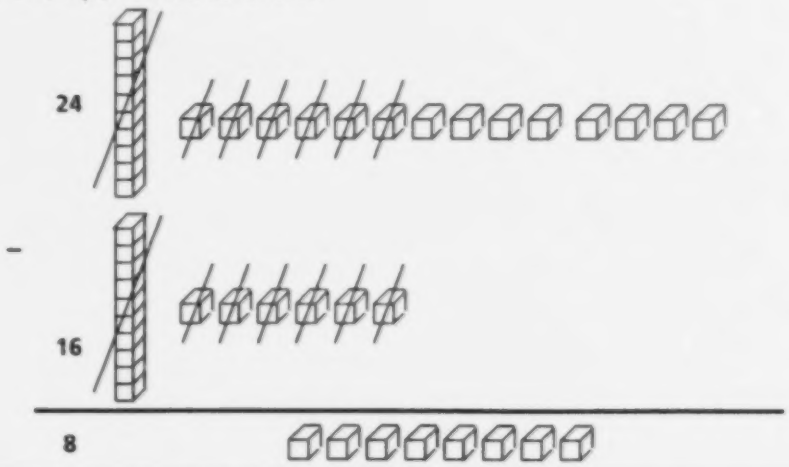
Regrouping in Subtraction



Note that there are not enough ones in the minuend to permit subtraction. It is necessary to regroup a "tens rod". The question becomes:

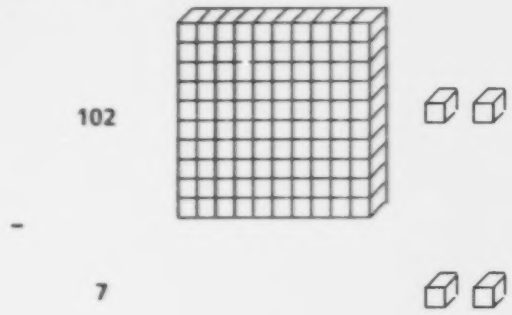


Subtraction is now performed as follows:

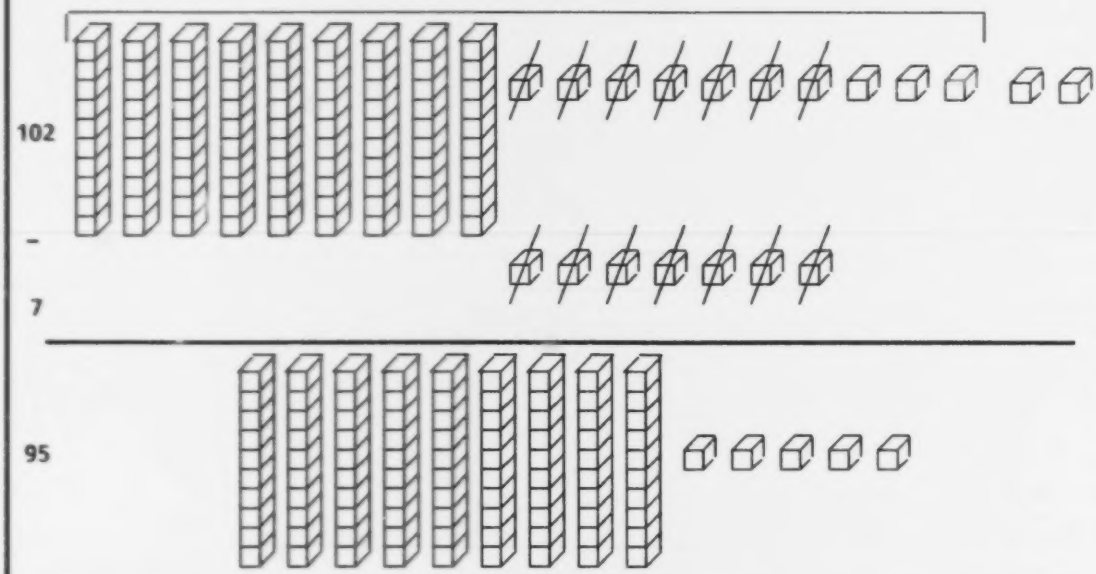


CLARIFICATION/EXAMPLE

Regrouping Over a Zero Digit



In order to subtract, it is necessary to regroup the "one hundred square". One hundred square is regrouped as:



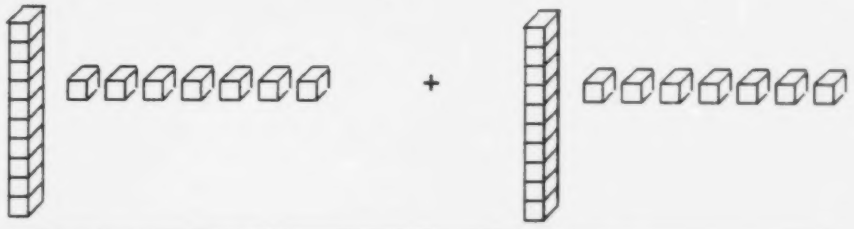
When regrouping in computational algorithms, encourage students to document the regrouping process in symbolic form:

$$\begin{array}{r} \overset{0}{\not 1}\ \overset{9}{\not 0}\ \overset{12}{\not 2} \\ -\quad\quad 7 \\ \hline 9\ 5 \end{array}$$

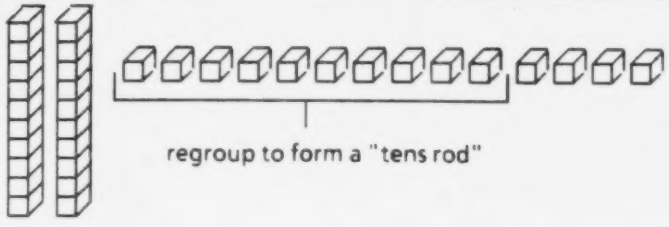
CLARIFICATION/EXAMPLE

Multiplication as Repeated Addition

17 × 2

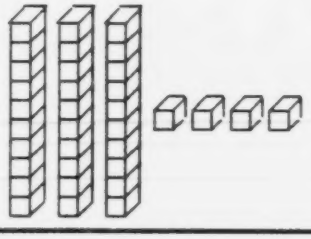


20 + 14



OR

34

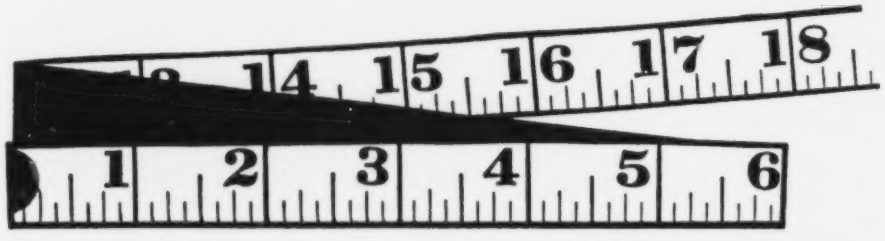


A tape measure can also be used to represent multiplication as a sequence of repeated additions.

CLARIFICATION/EXAMPLE

6 × 3 = ?

Locate the mark for 6 units on the tape measure. Fold the tape so that you have a total of three lengths of 6 units. The answer is the number that appears at the end of the last fold.



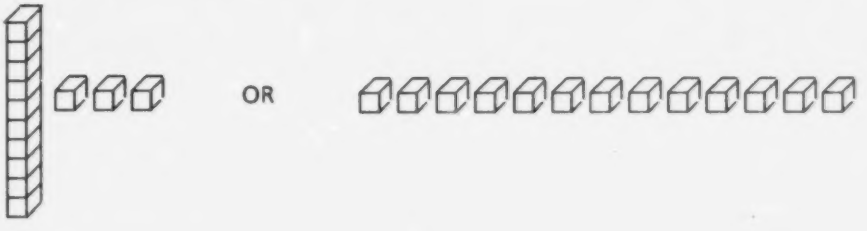
Students frequently experience difficulty with division. Encourage students to recognize division as a "sharing process". The sequence of questions provided below will give direction in the use of appropriate thought processes.

- How many are to be shared?
- How many are doing the sharing?
- Can we share so that everyone gets a ten? a hundred?
- What do you think is the greatest number each will have when the sharing is done?
- Are there any left over?
- Is there anything that we can use to help us decide the greatest number that can be shared?

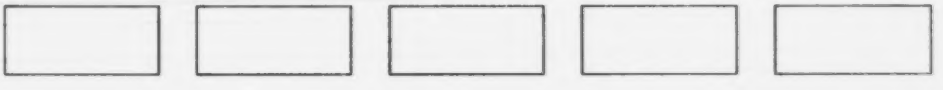
CLARIFICATION/EXAMPLE

13 ÷ 5 = ?

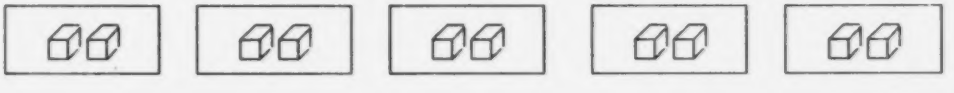
- How many are to be shared?



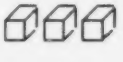
- How many are doing the sharing?



- What is the greatest number that each will have when the sharing is done?



- Are there any left over?

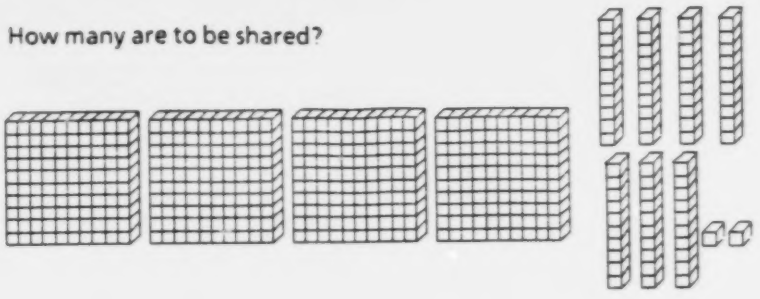


Therefore, 13 ÷ 5 = 2 R 3.

CLARIFICATION/EXAMPLE

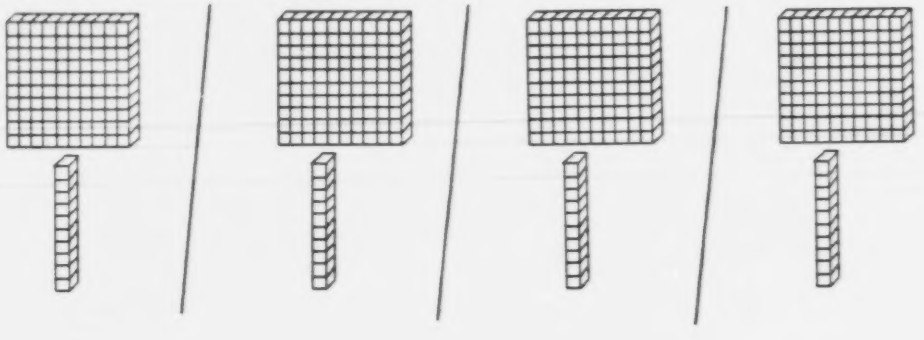
$472 \div 4 = ?$

- How many are to be shared?

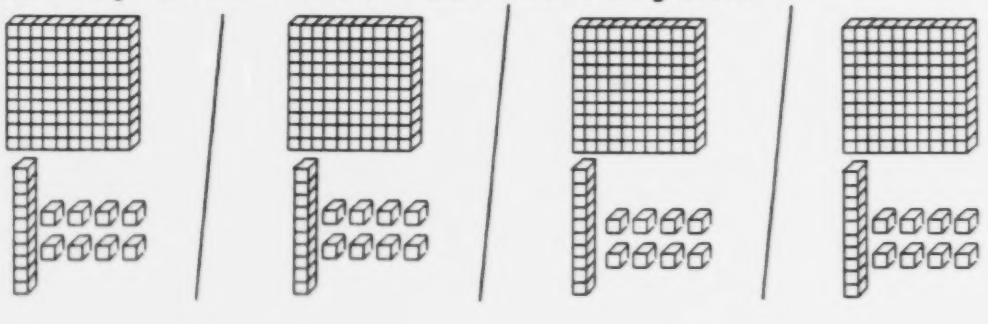


- How many are doing the sharing?

- Can we share so that everyone gets a hundred? a ten?



- What is the greatest number each will have when the sharing is done?



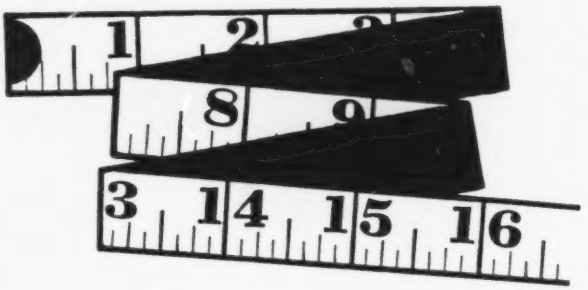
Therefore, $472 \div 4 = 118$ R 0.

Students who experience difficulty in understanding the division process will benefit from its representation in a variety of forms. The tape measure provides an alternative method of demonstrating the division process.

CLARIFICATION/EXAMPLE

16 ÷ 3 = ?

Locate the first 16 units on the tape measure. Starting at 16, fold this portion of the tape measure into sections that are 3 units in length.



The number of folded sections will represent the quotient. The number of units left beyond the last fold will represent the remainder.

Therefore, 16 ÷ 3 = 5 R 1.

Students often have difficulty with the traditional algorithm for division. If students find other algorithms more meaningful, encourage their use.

CLARIFICATION/EXAMPLE

Alternative Division Algorithms:

$$
\begin{array}{r|l}
4 & 978 \\
\hline
 & 400 \quad 100 \\
 & 578 \\
 & 400 \quad 100 \\
 & 178 \\
 & 160 \quad 40 \\
 & 18 \\
 & 12 \quad 3 \\
 & 6 \\
 & 4 \quad 1 \\
 & 2 \quad 244
\end{array}
\qquad
\begin{array}{r|l}
4 & 978 \\
\hline
 & 800 \quad 200 \\
 & 178 \\
 & 160 \quad 40 \\
 & 18 \\
 & 16 \quad 4 \\
 & 2 \quad 244
\end{array}
\qquad
\begin{array}{r|l}
 & 4 \\
 & 40 \\
 & 200 \\
\hline
4 & 978 \\
\hline
 & 800 \\
 & 178 \\
 & 160 \\
 & 18 \\
 & 16 \\
 & 2
\end{array}
\qquad
\begin{array}{r|l}
 & 244 \\
\hline
4 & 978 \\
\hline
 & 8 \\
 & 17 \\
 & 16 \\
 & 18 \\
 & 16 \\
 & 2
\end{array}
$$

DEVELOPING MENTAL ARITHMETIC SKILLS

Mental arithmetic and estimation have a reciprocal relationship in that the ability to perform simple mental calculations is basic to refining one's estimation skills, and some methods of estimation provide a framework for performing mental arithmetic. Nevertheless, there are two distinguishing characteristics of mental arithmetic:

- it produces an exact answer
- it is performed mentally, without the aid of external devices such as paper and pencil.

The following strategies will assist teachers in emphasizing the development of mental arithmetic skills throughout the mathematics program:

- Make a commitment to devote time each day to the development of strategies for mental arithmetic. Plan to teach specific strategies and offer opportunity for practice on a regular basis.
- Make a list of mental arithmetic strategies that you consider are most appropriate for your students. Encourage discussion and sharing of these strategies.
- Ask students to "think aloud" and to share the strategies that they already use. In addition to helping students to focus on their own strategies, this process may provide other students with new strategies for performing mental computation.
- Include the use of whole numbers, decimals and fractions in mental arithmetic activities. Develop strategies for performing various operations with these numbers.
- Provide frequent opportunities for students to practice mental arithmetic through oral activity. Emphasis on written computational work often tends to discourage the application of skills in mental computation.
- Develop an evaluation plan that reflects your instructional commitment to mental arithmetic. Regular tests on mental arithmetic skills that have been taught will remind students that you are serious about the importance of these skills.

RECALLING AND APPLYING BASIC FACTS

Provide opportunity at the beginning or end of each class for students to recall and apply basic number facts. Questions should be answered without the use of paper and pencil, and may involve:

- single digit number facts (e.g., 7×9)
- "extended" number facts (e.g., 7×90)
- sequences of operations (e.g., $5 \times 6 + 5 - 7$)
- multiplication and division by powers of ten.

Activities worthwhile in developing student ability to recall and apply basic number facts include:

- the number game "Krypto". This game can be played for short periods of time, and reinforces single digit number facts and sequences of operations.
- "mad-minute" drills. Timed challenges that become part of the daily routine will encourage recall of basic facts, as well as foster a habit for their use.
- the use of "graph-paper arrays" or "algebra tiles". Students may benefit from constructing arrays that illustrate the factors for "hard-to-remember" numbers.
- the use of computer programs. A variety of computer programs that reinforce basic facts are available for use (e.g., Fast Facts).

Encourage students to "extend" their basic number facts to related situations. This approach will allow some variation to practice sessions, and also help to develop number sense.

CLARIFICATION/EXAMPLE

- Extended Number Facts for "2 + 3":
  - 20 + 30
  - 20 000 + 30 000
  - 0.2 + 0.3.
- Extended Number Facts for "6 × 7":
  - 6 × 70
  - 60 × 70
  - 0.6 × 700.

Students should be encouraged to use strategies of "doubling" and "halving" to simplify mental computations.

CLARIFICATION/EXAMPLE

Use a doubling strategy to recall a forgotten multiplication fact, such as 12 × 7.

Step One: Take half of 12 and multiply by 7.
e.g., 6 × 7 = 42

Step Two: Double the result.
e.g., 42 × 2 = 84

Therefore, 12 × 7 = 84.

Factoring is another useful method of simplifying multiplication, thus enabling the student to find answers through the process of mental arithmetic.

CLARIFICATION/EXAMPLE

24 × 15 = ?

Step One: Factor the numbers.

24 X 15

= (4 X 6) X (5 X 3)

Step Two: Rearrange the factors.

(4 X 6) X (5 X 3)

= (4 X 5) X (6 X 3)

Step Three: Apply basic facts.

(4 X 5) X (6 X 3)

= 20 X 18

360

Discuss and display the following shortcuts for multiplying by 5, 25 and 50;

- To multiply a number by 5, multiply by 10 and divide by 2.
- To multiply a number by 25, multiply by 100 and divide by 4.
- To multiply a number by 50, multiply by 100 and divide by 2.

Encourage students to discuss and devise other shortcuts that may be worthwhile in performing mental arithmetic.

USING FORMAL STRATEGIES

Two formal strategies that are frequently used in performing calculations with mental arithmetic include:

- computing from left to right
- compensation.

Model and discuss the use of these strategies on a regular basis. Encourage students to use these or other worthwhile strategies in their daily work.

CLARIFICATION/EXAMPLE

STRATEGIES FOR ADDING WITH MENTAL ARITHMETIC

LEFT TO RIGHT

(4)8 + (3)5

1. Add the tens — (40 and 30 is 70)
2. Add the ones — (8 and 5 is 13)
3. Add the two sums — (70 + 13 = 83)

(48) + (3)5

1. Look at the whole first number. Add to it the tens in the second number. — (48 and 30 is 78)
2. Add the ones in the second number to the sum. — (78 + 5 = 83)

COMPENSATION

(48)$^{+2}$ + (35)$^{-2}$

1. Add a number to one addend to make it a multiple of ten. — (48 and 2 is 50)
2. Subtract the same number from the other addend — (35 − 2 is 33)
3. Add the two numbers. — (50 + 33 = 83)

(48) + 19 (for use in adding 19, 29, etc.)

1. To the first number add 20 (the next higher multiple of ten). — (48 and 20 is 68)
2. Subtract 1 from the sum. — (68 − 1 = 67)

CLARIFICATION/EXAMPLE

STRATEGIES FOR SUBTRACTING WITH MENTAL ARITHMETIC

LEFT TO RIGHT

(62) - (2) 5

1. From the first number subtract the tens in the second number.

   (62 minus 20 is 42)

2. Now subtract the ones in the second number.

   (42 – 5 = 37)

COMPENSATION

+5 +5

(62) - (25)

1. Add a number to the minuend to make it a multiple of ten.

   (25 and 5 is 30)

2. Add the same number to the subtrahend.

   (62 + 5 = 67)

3. Subtract the two numbers.

   (67 – 30 = 37)

CLARIFICATION/EXAMPLE

STRATEGIES FOR MULTIPLYING WITH MENTAL ARITHMETIC

LEFT TO RIGHT

[6 × 72]

1. Multiply the tens in the second factor by the first factor. (6 × 70 = 420)
2. Multiply the ones in the second factor by the first factor. (6 × 2 = 12)
3. Add the two products. (6 × 72 = 432)

COMPENSATION

[6 × 39]

1. Round the second factor to the next higher ten. (39 → 40)
2. Multiply the tens by the first factor. (6 × 40 = 240)
3. Multiply the difference by the first factor. (40 – 39 = 1; 6 × 1 = 6)
4. Subtract the second product from the first one. (240 – 6 = 234)

DEVELOPING STRATEGIES FOR ESTIMATION

Computational estimation has four defining characteristics:

- it is usually done mentally, without the aid of paper and pencil
- it is done quickly
- the answers are not exact
- it reflects individual approaches.

Estimation should be frequently practiced in a variety of ongoing activities. The guidelines which follow may be useful in creating a classroom climate that establishes the usefulness of estimation and builds student confidence in making estimates.

- Introduce estimation with examples where estimated or rounded counts are used in real life. Extend this activity by having students identify numbers used in the newspaper that are exact amounts and those that are estimates:
  - "Over 1500 Fans View Final Home Game"
  - "$200 000 Spent on Recreational Centre"
  - "2/3 of the Athletes Use Brand X".
- Emphasize situations where only an estimate is required. Estimation is useful both as a check on the reasonableness of a computed answer and as an end in itself. Many daily situations require only an estimate.
- Use easy examples in early stages of instruction, and avoid requiring too much precision in estimates. Students need to be convinced that estimation is easy and develop a desire to use it.
- Emphasize the language of estimation. The use of appropriate phrases will communicate the spirit of estimation and help students to understand it:
  - about 12 1/2
  - close to 9
  - just about 15
  - a little less than 3.5
  - between 8 and 9, but probably closer to 8
  - somewhere between 30 and 40.
- Accept a variety of answers. Students need to understand that there is no one "correct" estimate. Any estimate that is reasonably close to the exact answer is valid. If multiple responses are encouraged, students will learn more about the process of estimation.
- Use oral work and group discussion. In the early stages of instruction, students will be tempted to compute the exact answer and adjust it by rounding. By encouraging oral response, anxiety levels will be reduced and students will be more likely to apply estimation strategies. Oral work promotes "doing estimation in your head" rather than with paper and pencil.
- Present situations in which students must decide what type of estimate is required. Estimates can range from rough to precise. Sometimes only a ball-park estimate is required, but at other times it is important to get closer to the exact answer. Discussion should emphasize that both ball-park estimates and more precise ones are appropriate, and the choice of which one to use depends on the situation.
- Have students identify the number of digits that an answer to a specific computational question should contain. Focusing on the size of the answer provides a quick and useful check for reasonableness.
- Present examples where students must:
  - identify estimates that are unreasonable
  - show whether an estimate is an overestimate or an underestimate
  - adjust an initial estimate by changing it to a closer estimate.

- Evaluate student progress in estimation. Use an overhead to project a variety of problems. Project each problem individually for a short period of time (fifteen to twenty seconds). Scoring intervals can be set up in advance for each problem. Be sure to take time to discuss selected problems and the strategies that students have used to solve them.

A computer program in BASIC that will provide students with an opportunity to practice their skills in estimation has been included as Resource 1: Estimation Game.

FRONT-END ESTIMATION

This method of estimation is useful in addition and subtraction with numbers of three or more digits. The left-hand digits are added or subtracted. An estimate for the remaining part of the problem is then determined by "eyeballing" the remaining digits.

```
  426
  275
+ 126
```

a. Add the hundreds → 400 + 200 + 100 = 700
b. Estimate the sum of tens and ones → greater than 100 but less than 200
c. Estimate the total → between 800 and 900

```
  626
- 147
```

a. Subtract the hundreds → 600 – 100 = 500
b. Compare 26 and 47 → 26 is less than 47
c. Estimate the difference → The answer is between 400 and 500.

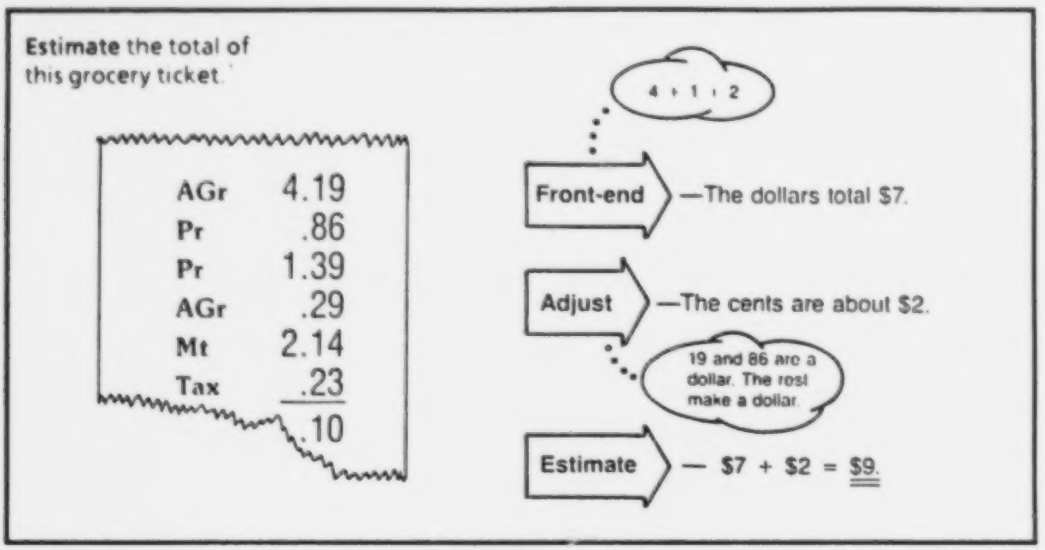
The front-end method can also be used for some multiplication questions.

$4 \times 648 = \square$

a. $4 \times 600 = 2400$
b. 48 is about 50
c. $4 \times 50 = 200$
d. the product is about 2600

One advantage of the front-end method of estimating is that it is less likely to require the use of paper and pencil than other methods.

CLARIFICATION/EXAMPLE



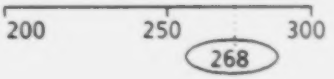
ROUNDING

The concept of rounding numbers needs careful development before students can apply it effectively to estimating answers. Students need to understand the thought processes and steps employed in rounding a number. The illustration below uses a number line in developing the concept of a rounded number. Students should relate this process to all rounding situations.

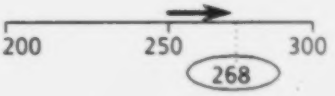
1. Round 268 to the nearest hundred.

2. Determine between which hundreds the number lies.

3. Find the "midpoint" number.



4. Determine which hundred the number is closer to by noting whether it is greater or less than the midpoint number.



5. Write the rounded number.

Although rounding is the most familiar approach to estimating, it should not be used to the exclusion of other approaches.

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[1] Reprinted with permission from *The Arithmetic Teacher*. *Copyright* 1985 by the National Council of Teachers of Mathematics.

CLARIFICATION/EXAMPLE

Example		*Estimate*
362 + 825	→	400 + 800
49 × 6	→	6 × 50
68 × 32	→	70 × 30
1782 ÷ 3	→	1800 ÷ 3

COMPATIBLE NUMBERS

Rounding and the use of front-end numbers do not always result in easy mental computation. By rounding, $17.6\overline{)338}$ becomes $18\overline{)340}$. By using front-end numbers, $17.6\overline{)338}$ becomes $17\overline{)330}$. Neither method results in an easy division. Consider, instead, the possibilities of $17\overline{)340}$ or $18\overline{)360}$. Each of these division questions has a quotient that is easy to calculate mentally.

This strategy of estimating with numbers that are easy to calculate mentally is called "estimating with compatible numbers". What constitutes compatible numbers depends upon the operation as well as the estimator's choice.

The compatible numbers strategy can also be used with addition, subtraction and multiplication. As illustrated below, the student looks for pairs of numbers that "fit together" to make numbers that are easy to compute mentally.

14.83
7.14
25.82
3.41

approximately 15 + 25 = 40
approximately 3 + 7 = 10

40 + 10 = 50

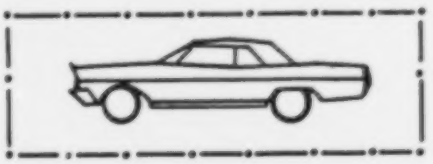
Model this estimation strategy, and provide frequent opportunity for students to apply the strategy to their work.

CLARIFICATION/EXAMPLE

Estimate the monthly payments.[1]

Financed cost: $15 629

Payments: 48 months



The problem is easier to think of as

$50\overline{)15\,000}$

Compatible number estimate: $300

CLUSTERING (OR AVERAGING)

The clustering or averaging strategy is suited for a particular type of problem that is often encountered in everyday situations. It can be used when a group of numbers cluster around a common value. To be successful in using this strategy, students should understand the concept of "average", and recognize the link between repeated addition and multiplication. Although clustering is limited to a certain type of problem, the strategy is useful. Many students will discover and use this strategy on their own.

CLARIFICATION/EXAMPLE

Estimate the total attendance.[2]

Monday	72 250
Tuesday	63 891
Wednesday	67 490
Thursday	73 180
Friday	74 918
Saturday	68 490

The figures all cluster around 70 000, so about 70 000 people attended each day.

$6 \times 70\,000 = 420\,000$

Averaging estimate: 420 000

[1] Reprinted with permission from *The Arithmetic Teacher*. Copyright 1985 by the National Council of Teachers of Mathematics.

[2] Ibid.

RESOURCE 1: ESTIMATION GAME[1]

The BASIC program below provides students with an opportunity to practise their skills in estimation. The program involves addition, but can easily be modified to provide practice with any of the operations. It will work with both whole numbers and decimals.

Players take turns entering a value of their choice. Each player must then make an estimate of the results of adding the numbers that have been entered. The estimate closest to the actual answer is declared the winning estimate.

```
  4  PRINT "ESTIMATION GAME"
 10  LET OP$ = "SUM"
 15  PRINT "IN TURN, EACH PLAYER WILL ENTER"
 20  PRINT "A NUMBER OF THEIR CHOICE"
 30  PRINT "THEN EACH PLAYER WILL ENTER AN"
 35  PRINT "ESTIMATE THE OP$ OF THE NUMBERS"
 45  PRINT "THE CLOSEST ESTIMATE WINS"
 55  PRINT "FIRST PLAYER'S NUMBER": INPUT A
 60  PRINT "SECOND PLAYER'S NUMBER": INPUT B
 70  PRINT "FIRST PLAYER'S ESTIMATE...": INPUT A1
 80  PRINT "SECOND PLAYER'S ESTIMATE..." : INPUT B1
 85  LET C = A + B: REM FINDS THE SUM
 90  IF ABS (C - A1) < ABS (C - B1) THEN 115
 96  IF ABS (C - A1) = ABS (C - B1) THEN 117
100  PRINT "PLAYER #2 WINS!"
105  GOTO 130
115  PRINT "PLAYER #1 WINS!"
116  GOTO 130
117  PRINT "IT'S A TIE!"
130  PRINT "PLAY AGAIN? (Y OR N)"
131  INPUT AN$$
132  IF AN$$ = "Y" THEN GOTO 55
133  IF AN$$ < > "N" THEN GOTO 130
140  END
```

Line 10 can be changed to indicate a different operation. By changing line 10 to LET OP$ = "PRODUCT", the instructions in line 35 will tell the user to estimate the answer to a multiplication problem. If line 10 is changed, then the operation in line 85 must also be changed. If OP$ = "PRODUCT", then line 85 must say LET C = A*B.

[1] Alberta Education, Curriculum Support, *Junior High Mathematics Teacher Resource Manual*, 1988.

USING A MATH LAB

The purpose of a math lab is best illustrated by an ancient Chinese proverb:

"I hear and I forget
I see and I remember
I do and I understand".

Although students are at various stages of cognitive development, most will continue to use concrete operational thinking. Instruction should begin at the concrete level, with emphasis placed on experiential learning that involves the use of manipulatives and hands-on activities. A thoughtfully designed math lab will provide opportunity for students to use a variety of tactile and visual materials. As students use these materials, they should be encouraged to observe, verbalize and discuss the relationships being investigated, and to translate these relationships into symbolic and abstract forms.

There are many commercially produced math kits available, but these can be expensive and may contain items that are not really needed. It is suggested that teachers set up their math labs using items they have borrowed, collected, purchased or made. The lab may contain:

- assorted measuring devices
  - tape measures/rulers/metre sticks
  - measuring cups
  - graduated cylinders
  - scales for measuring mass/weight
  - assorted thermometers
  - protractors
- set of cuisenaire rods
- base-ten blocks
- paper money and plastic coinage
- banking forms
- cash register tapes and order forms
- graph paper
- geoboards
- tangram pieces
- fraction circles
- LOGO computer program
- other manipulative materials such as popsicle sticks, coloured paper, buttons.

The activities that support an experiential approach to mathematics and involve the use of manipulative materials which have been included in this section of the manual are:

- Number Systems and Operations
- Ratio, Proportion and Percent
- Geometry and Measurement
- Project Work.

NUMBER SYSTEMS AND OPERATIONS

WHOLE NUMBERS AND DECIMALS

Base-ten blocks provide a concrete representation of arithmetical operations and process with both whole numbers and decimals. A description of their use is included in the "Computational Facility and Estimation" section of this manual.

Puzzles and games also provide a worthwhile method of reinforcing arithmetical process and basic number facts.

1. BUZZ

Have a group of students play the game of Buzz. Students count from one in a clockwise/counterclockwise direction. Each time the number seven or a multiple of seven is reached, the student must say "Buzz" instead of the number. If a student makes an error and says the number instead of "Buzz", that student is out of the game. The counting starts over again with the next player. The last player left in the game is the winner.

Note:
This game reinforces basic multiplication facts. The teacher may vary the rules to this game, and choose numbers other than seven as the subject of the game.

2. LETTER PUZZLES[1]

Students may benefit from designing/solving letter puzzles in addition or subtraction.

In the following puzzle, each letter represents a digit:

```
  SEND
+ MORE
------
 MONEY
```

The solution is:

```
  9 5 6 7
+ 1 0 8 5
---------
1 0 6 5 2
```

Letter puzzles may also be used to develop and reinforce problem-solving strategies and number properties. In the example that follows, each letter again represents a digit:

```
  YZ
x  2
----
 XZZ
```

Ask students to identify the number, which when doubled, yields itself. Students will recognize the multiplicative property of zero, and should then be able to solve the rest of the puzzle.

The solution is:

```
  50
x  2
----
 100
```

[1] Alberta Education. *Let Problem Solving Be The Focus For The 1980's*, 1983.

Another problem:

$$\begin{array}{r} VZVZ \\ \times \quad 2 \\ \hline X2X2Z \end{array}$$

The solution is:

$$\begin{array}{r} 6060 \\ \times \quad 2 \\ \hline 12120 \end{array}$$

Relate the use of letters in these puzzles to the use of variables in algebra.

3. DOMINOES

Percent-Decimal Dominoes (2 players)

Shuffle the deck (20 cards) and place cards face down on the table. Each player takes 4 cards from the table.

Play alternates.

Player 1 begins play by placing a domino face up on the table.

Player 2 plays a domino on any edge of the first domino, making sure that the touching edges name the same number. If unable to play, the player draws one card from the face-down cards and plays it if possible. If it is not possible to play, the other person takes a turn.

The first person to play all of their cards wins the game.

Note:
The game may be extended to more than one dealing. Score one point for each card left at the end of each game. The first person to reach 10 points loses the game.

0.07	13%	0.2	80%	0.02	4%	0.9	60%
0.13	8%	0.8	70%	0.04	50%	0.6	31%
0.08	40%	0.7	18%	0.5	6%	0.31	9%
0.4	0.05	0.18	3%	0.06	10%	0.09	1%
5%	20%	0.03	2%	0.1	90%	0.01	7%

Money Dominoes (2 players)

Shuffle the deck of 20 cards and place cards face down on the table. Each player takes four cards from the table.

Play alternates.

Player 1 begins play by placing a domino face up on the table.

Player 2 plays a domino on any edge of the first domino, making sure that the touching edges name the same number. If unable to play, the player draws one card from the face-down cards and plays it if possible. If it is not possible to play, the other person takes a turn.

The first person to play all of their cards wins the game.

Note:
The game may be extended to more than one dealing. Score one point for each card left at the end of each game. The first person to reach 10 points loses the game.

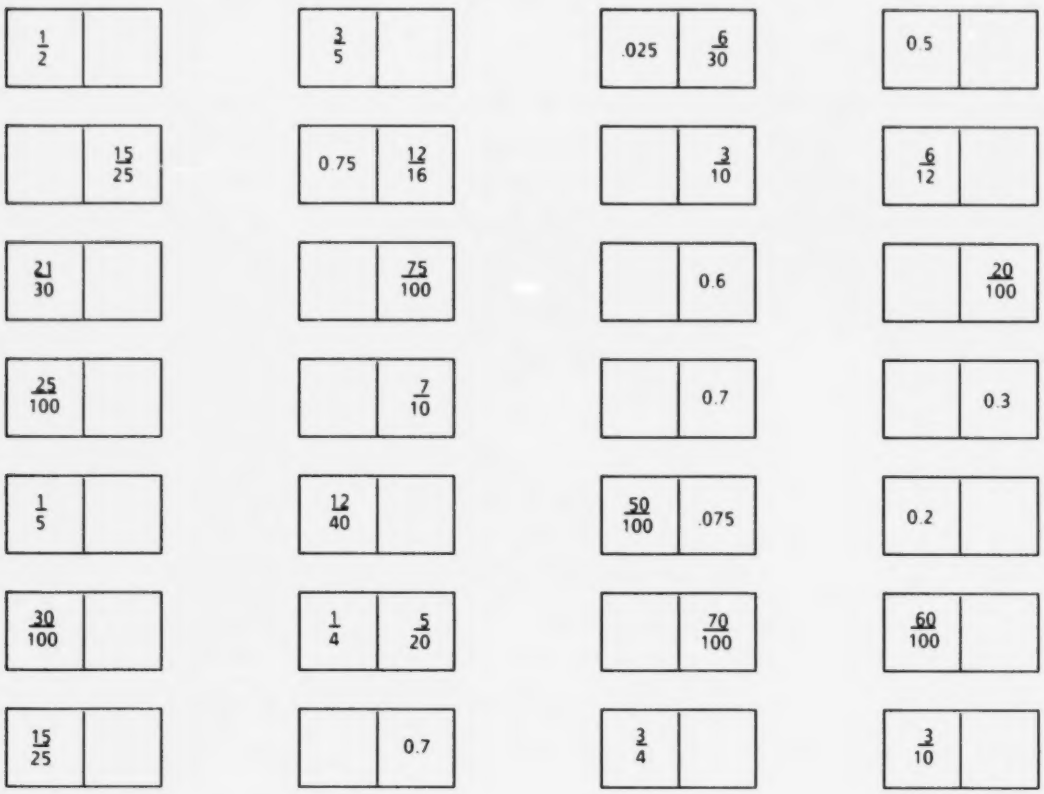
1 dime	4/100
4 pennies	50/100
2 quarters	35/100
7 nickels	1/100
1 penny	30/100
3 dimes	3 quarters
75/100	6 dimes

60/100	21 pennies
21/100	4 nickels
20/100	4 dimes
40/100	1 nickel
5/100	90/100
9 dimes	25/100
1 quarter	55/100

11 nickels	3/100
3 pennies	79/100
79 pennies	70/100
7 dimes	15/100
3 nickels	6/100
6 pennies	10/100

Fraction – Decimal Dominoes (4 players)

The rules are similar to those for Money Dominoes, except 28 cards are used and each player starts with seven dominoes (cards). Play goes around the table in a clockwise direction.



4. PERCENT – DECIMAL PUZZLE

Distribute one copy of this grid to each student in the class. Instruct students to cut out the 16 pieces and assemble the puzzle so that touching edges name the same number (i.e., decimal-percent equivalents).

The finished puzzle will be a 4 x 4 square.

Note:
Other variations of this puzzle are easily made using a 3 x 3, 4 x 4 or 5 x 5 grid.

83% 0.61 0.1 40%	0.04 1% 52% 0.72	0.03 5% 0.8 0.13	23% 0.05 20% 0.79
70% 80% 0.29 0.38	9% 10% 16% 3%	0.61 39% 5% 0.32	0.6 29% 33% 4%
0.06 0.18 8% 61%	72% 17% 0.52 0.31	49% 0.02 81% 60%	13% 0.08 25% 0.23
0.4 0.39 0.05 6%	38% 0.25 0.01 9%	0.07 0.2 0.17 30%	27% 0.16 2% 0.7

5. MAGIC SQUARES

Place the numerals 1 through 9 in a square like the one below so that you may add the three numerals in any direction and the sum will always be 15.

Note:
Magic squares may also be constructed using decimals and fractions:

- $\frac{1}{10}$, $\frac{1}{5}$, $\frac{3}{10}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{3}{5}$, $\frac{7}{10}$, $\frac{4}{5}$, $\frac{9}{10}$ (the sum in any direction will be $1\frac{1}{2}$)
- 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45 (the sum in any direction will be 0.75).

INTEGERS

Integer concepts should be developed through a variety of real life applications. Provide opportunities for students to demonstrate addition of simple pairs of integers through concrete manipulation and diagramatic representation.

1. Positive and negative integers can be represented to students through the analogy of having money (positive integers) and owing money (negative integers).

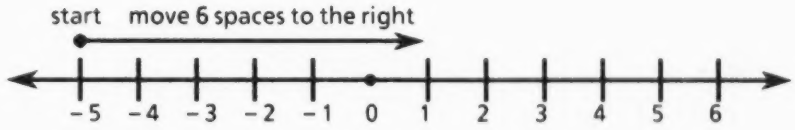
CLARIFICATION/EXAMPLE

Frank makes $7.00 cutting the lawn. He uses this money to pay back a debt of $10.00 which he owes his father. How much money does Frank owe his father after repaying him $7.00?
$-10 + 7 = -3$

2. Integer addition may be represented as moves on the number line. Moves to the right are positive, while moves to the left are negative.

CLARIFICATION/EXAMPLE

$-5 + 6 = ?$



Start at −5. Because 6 is positive, move 6 spaces to the right.
$-5 + 6 = 1$

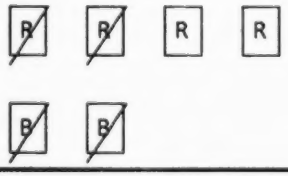
3. Use the thermometer to demonstrate integer addition in the same way that the number line is used. Positive integers represent a rise in temperature, while negative integers will represent a drop in temperature.

4. Coloured cards may be used to demonstrate the addition of integers. Represent positive integers with cards of one colour (e.g., black), and negative integers with cards of another colour (e.g., red).

CLARIFICATION/EXAMPLE

Black cards are positive.
Red cards are negative.

$-4 + 2 = ?$



Pairs of red and black cards cancel each other.

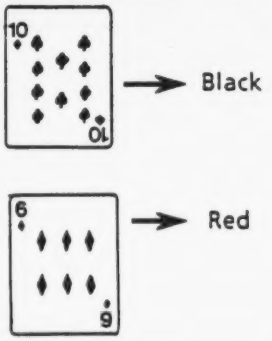
The answer is two red or −2.

Integer addition can also be demonstrated using a regular deck of playing cards. Represent positive integers with black cards and negative integers with red cards. Pairs of red and black cards with the same number will cancel each other.

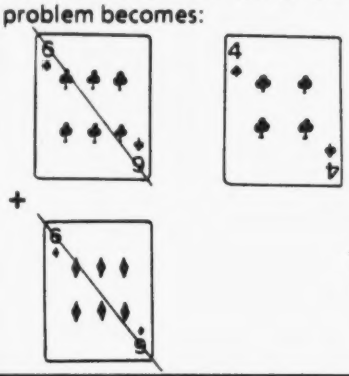
CLARIFICATION/EXAMPLE

$10 + -6 = ?$

Initially, the problem could be represented as:



These cards do not cancel each other. Replace the 10 with a six and a four. The problem becomes:

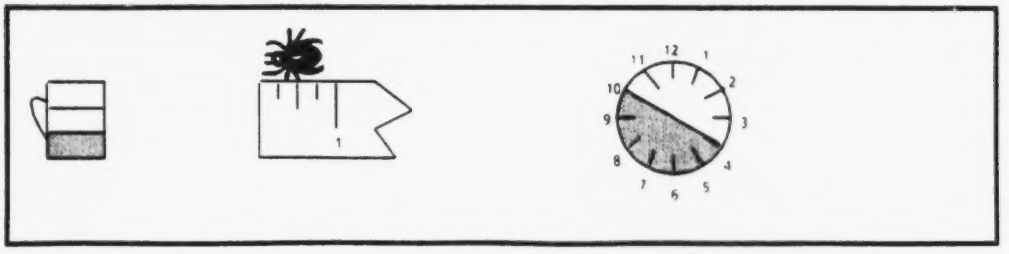


The answer is a black 4.
$10 + -6 = 4$

FRACTIONS

1. Develop the concept of a fraction through the use of objects, pictures and diagrams. Plan activities that include concrete manipulation and pictorial representation. Describe patterns and relationships with numbers and symbols.

CLARIFICATION/EXAMPLE



2. Fraction circles provide a useful method of developing the concept of a fraction and an understanding of equivalence among fractions. A variety of blackline masters for producing fraction circles have been provided as Resource 1: Fraction Circles.

CLARIFICATION/EXAMPLE

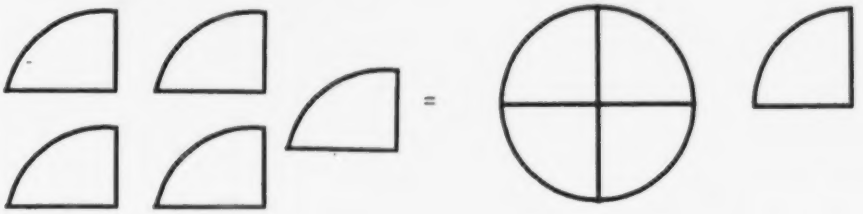
- Draw a diagram that illustrates the relationship between 4/4 and 1.

$\frac{4}{4} = 1$

- Draw the diagram that illustrates the relationship between 2/4 and 1/2.

$\frac{2}{4} = \frac{1}{2}$

- Investigate the relationship between improper fractions and mixed numbers using pieces of a fraction circle.



$\frac{5}{4}$ = $1\frac{1}{4}$

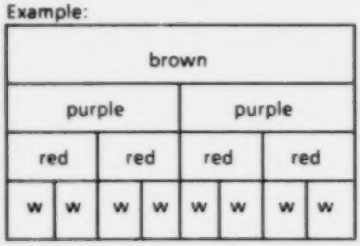
3. Cuisenaire rods provide another opportunity for developing an understanding of fractions in the tactile and visual mode. Teachers unfamiliar with the use of cuisenaire rods are encouraged to reference related instructional materials obtained from local media centres or from commercial publishers.

CLARIFICATION/EXAMPLE

DEVELOPING FRACTION CONCEPTS

A variety of activities similar to the one illustrated below can be used in developing fraction concepts.

Example:

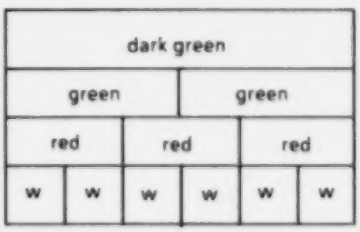


Record:

Colour of train	Number of rods	Fraction sentence
a) purple	2	$\frac{1}{4}$ of brown = red.
b) red	4	$\frac{1}{2}$ of brown = purple.
c) white	8	$\frac{1}{8}$ of brown = white.

RATIOS AND EQUIVALENT FRACTIONS

Cuisenaire rods provide opportunity for manipulative and visual experience in working with equivalent fractions and ratios.



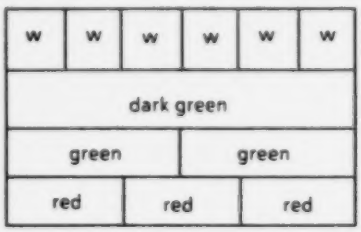
1) $\frac{\text{green}}{\text{dark green}} = \frac{1}{2} = \frac{3}{\square}$

2) $\frac{\text{red}}{\text{dark green}} = \frac{1}{\square} = \frac{\square}{6}$

3) $\frac{\text{white}}{\text{dark green}} = \frac{\square}{6} = \frac{\square}{12}$

ADDITION AND SUBTRACTION OF FRACTIONS

Cuisenaire rods will provide concrete support for the process of adding and subtracting fractions with like/unlike denominators.



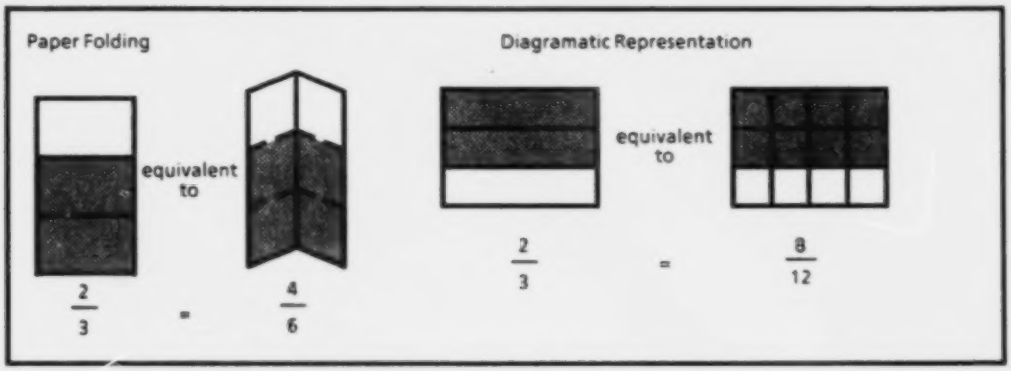
$\frac{1}{3} + \frac{1}{6}$

red + white = 3 whites or green

$\frac{1}{3} + \frac{1}{6} = \frac{3}{6}$ or $\frac{1}{2}$

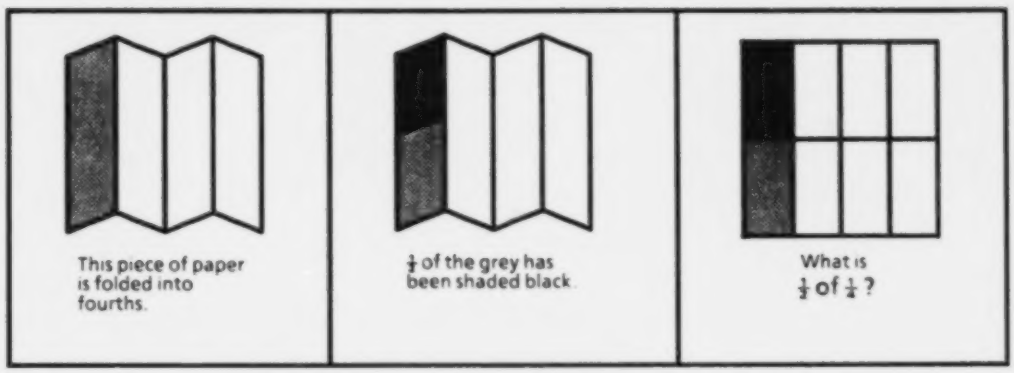
4. The concept of equivalent fractions (ratios) can be developed through paper folding activities, or through diagramatic representation with graph paper/geoboards. Encourage students to translate the relationships discovered into number sentences, and to look for patterns in the ratios obtained.

CLARIFICATION/EXAMPLE



Students can also investigate multiplication of fractions through paper folding activities. Encourage students to discover their own strategies for multiplying fractions that are based on the patterns and relationships discovered.

CLARIFICATION/EXAMPLE



5. The multiple board is a worthwhile tool for investigating equivalent fractions.

CLARIFICATION/EXAMPLE

To generate equivalent fractions using the multiple board, locate the original fraction in the first column. Numbers that are in the same position throughout the nine remaining columns will represent fractions that are equivalent to the original fraction.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

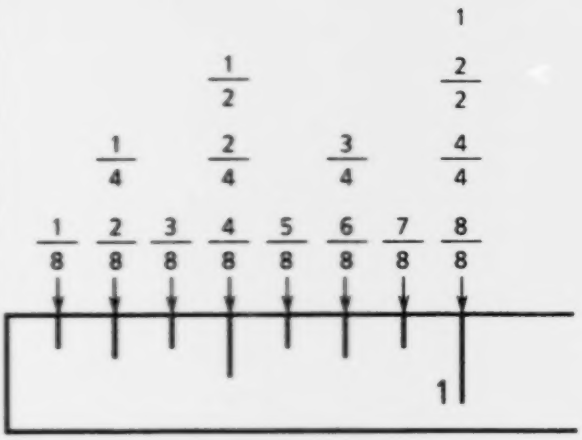
6. The "inch" unit provides a useful vehicle for developing/reinforcing fraction concepts. Design activities involving fractional parts of an inch that will develop the concepts illustrated below. A black line master of the inch unit is provided as Resource 2: Using the Inch Unit.

CLARIFICATION/EXAMPLE

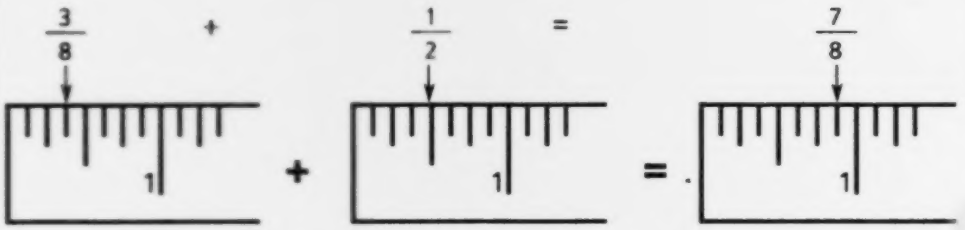
Order of Fractions on the Number Line

CLARIFICATION/EXAMPLE

Equivalent Fractions



Addition and Subtraction of Fractions

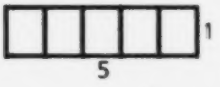


PRIME NUMBERS

1. The concept of "prime" is abstract, and should be developed using concrete ideas. The following strategy may be worthwhile in developing an understanding of prime numbers:
   (a) Provide students with a set of square blocks or square tiles. (Pieces of square paper will be sufficient.)
   (b) Select a number of blocks (square tiles) equal to a number which is to be factored.
   (c) Form as many rectangles as possible with this number of blocks (square tiles). Record your results.
   (d) The number being factored is prime if only one rectangle can be formed with the given number of blocks (square tiles).

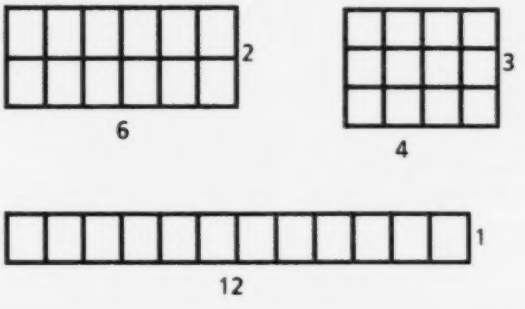
CLARIFICATION/EXAMPLE

- Factor the number "5":



The factors are 1 and 5. Since only one rectangle was formed, the number "5" is prime.

- Factor the number "12":



The factors are 1, 2, 3, 4, 6, and 12. Since more than one rectangle was formed, the number "12" is not prime.

2. Students may not recognize the importance of being able to identify factors and prime numbers. Encourage students to practise these skills by designing games that require their use.

CLARIFICATION/EXAMPLE

List the numbers from 1 to 50.

~~1~~	~~2~~	~~3~~	4	5	~~6~~	7	8	~~9~~	10
11	12	13	14	15	16	17	~~18~~	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

The first player selects a number (e.g., 18) and crosses it out. The second player crosses out all the factors of 18 and scores their total (e.g., 1 + 2 + 3 + 6 + 9 = 21). This process is then repeated, with the second player selecting a number and the first player crossing out its factors. Once a number has been crossed out it cannot be used again.

Exchange turns until all numbers have been crossed out. The player with the highest point total is declared the winner.

3. Computer programs can also be used by students to generate sets of factors for given numbers.

CLARIFICATION/EXAMPLE

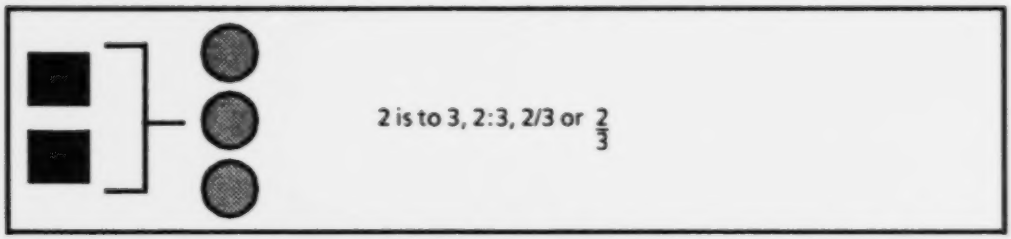
Basic Program

```
 10  PRINT "WHAT IS THE NUMBER"
 20  PRINT "YOU WISH TO FACTOR?"
 30  INPUT N
 40  PRINT "THE FACTORS ARE:"
 50  FOR K = 1 TO N STEP 1
 60  LET X = N/K
 70  LET Y = INT (N/K)
 80  IF X = Y THEN PRINT K
 90  NEXT K
100  END
```

RATIO, PROPORTION AND PERCENT

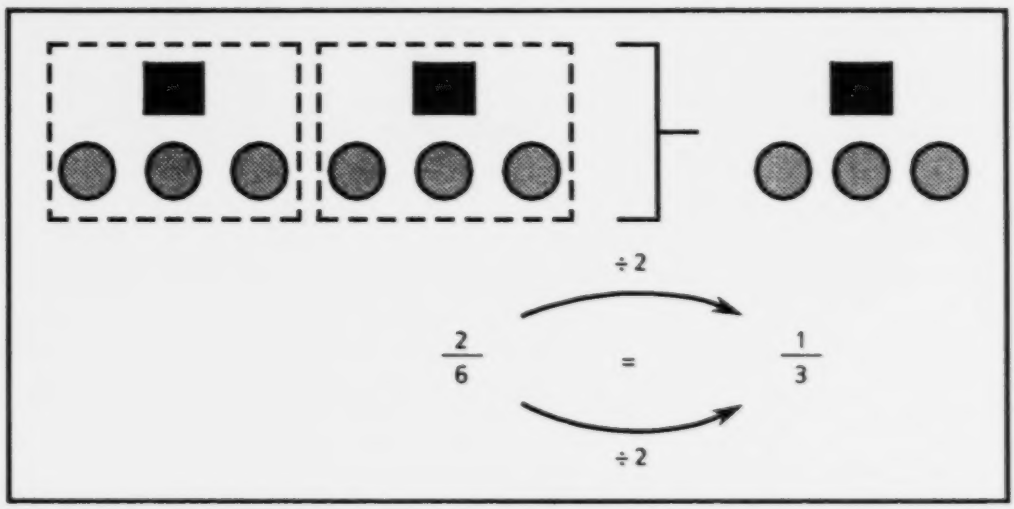
1. Provide opportunities for students to write ratios that describe and compare collections of <u>real objects</u> (e.g., blocks, coins, pieces of coloured paper, buttons). Students should use the three forms for expressing ratios interchangeably when describing their collections:
   - "a" is to "b"
   - a:b
   - a/b
   - $\frac{a}{b}$

CLARIFICATION/EXAMPLE



2. Encourage students to discover number relationships that are present in equivalent ratios by mapping a correspondence between objects in one collection and objects present in another collection. Through discussion, develop an understanding of how the equivalence of two ratios can be verified using common factors or multiples.

CLARIFICATION/EXAMPLE

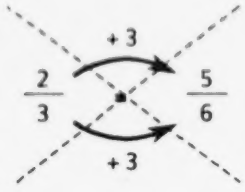


3. Students frequently fail to understand that the relationship between the two parts of a ratio is multiplicative rather than additive. The multiplicative relationship in ratios must be frequently illustrated and explained to students.

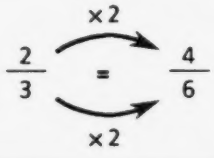
CLARIFICATION/EXAMPLE

$\frac{2}{3} = \frac{?}{6}$

Students may incorrectly reason that:



Emphasize the multiplicative relationship:

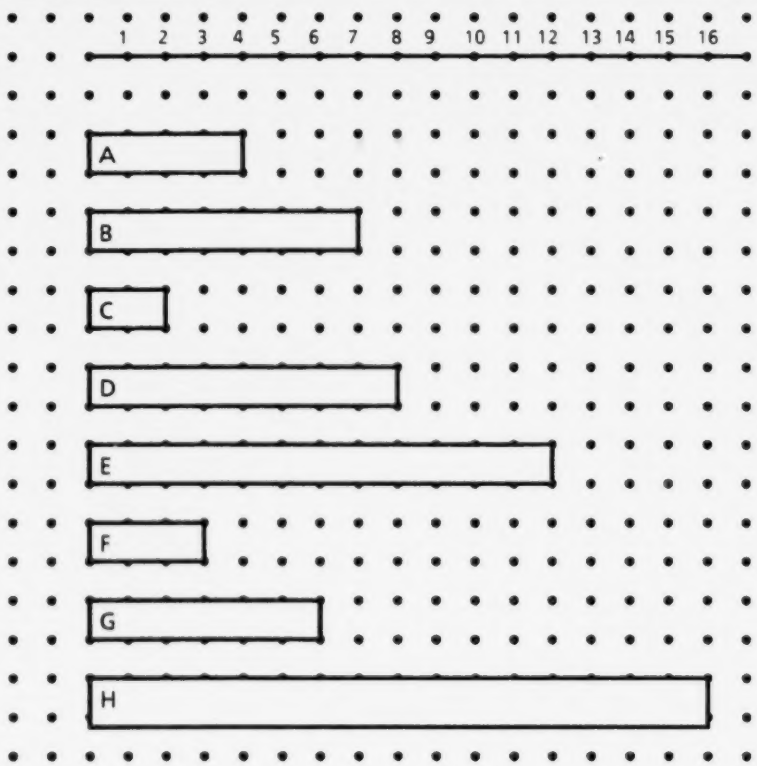


Students who continue to experience difficulty with this concept should be given additional experience in working with collections of real objects, and in mapping a correspondence between objects in one collection with objects in another.

4. Cuisenaire rods provide an effective means for developing ratio concepts through concrete experience. Teachers unfamiliar with the use of cuisenaire rods are encouraged to reference related instructional materials available from local media centres or from commercial publishers.

   The activity illustrated on the next page uses cuisenaire rods to provide a visual base for making comparisons with ratio. Similar activities are provided in Resource 3: Introduction to Ratios, and in Resource 4: Equal Ratios.

CLARIFICATION/EXAMPLE



Write the ratios of the following rectangles in two ways:

1. C to G		
2. B to D		
3. F to G		
4. G to H		
5. H to D		
6. A to G		
7. A to F		
8. D to G		

9. C to F		
10. B to G		
11. G to F		
12. H to B		
13. E to D		
14. A to D		
15. E to H		
16. G to D		

5. Keep number relationships simple when working with ratio and proportion. When applying ratio concepts to practical problems, select problem situations that involve both integral and non-integral ratios:

   - $\frac{1}{3} = \frac{4}{N}$ (integral)

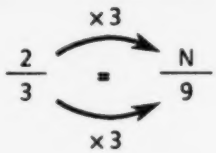
   - $\frac{4}{5} = \frac{N}{15}$ (non-integral).

6. Discuss the use of certain words in making comparisons that can be described with ratios:
   - six books for <u>each</u> student
   - six books <u>per</u> student
   - six <u>for a</u> dollar.

7. When finding the missing component in a proportion, ensure that students use the common multiple/common factor methods. Do not encourage the use of cross products at this level of development.

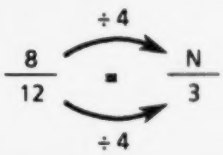
CLARIFICATION/EXAMPLE

Common Multiple Method



Numerator and denominator are multiplied by 3.

Common Factor Method



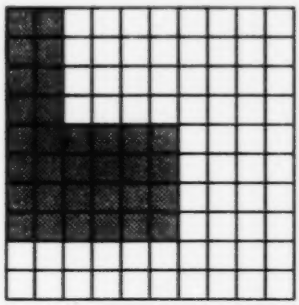
Numerator and denominator are divided by 4.

8. Develop the concept of percent through visual representation on the "100 grid". Ask students to illustrate given percents (and their equivalent ratios) by drawing and shading on a piece of 10 x 10 grid paper. A blackline master for the "100 grid" is provided in Resource 5: The 100 Grid.

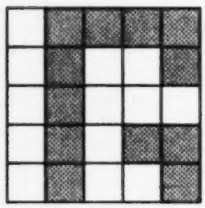
CLARIFICATION/EXAMPLE

DEVELOPING THE CONCEPT OF "PERCENT"

Provide concrete experiences that will enable students to conceptualize the meaning of percent. Begin with diagrams containing 100 sections, and then proceed to diagrams having other than 100 sections.



$\frac{32}{100} = 32\%$



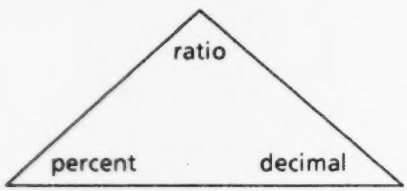
$\frac{12}{25} = \frac{48}{100} = 48\%$

9. Use a variety of procedures to reinforce understanding of common fraction, decimal and percent equivalents:
   - number line activities
   - fraction-decimal-percent dominoes.

   Triangular drill cards similar to the one illustrated below will be useful in developing student ability to recall equivalents that are commonly used.

CLARIFICATION/EXAMPLE

Construct a set of triangular cards. The vertices of each card will name a ratio, its equivalent decimal, and its equivalent percent.



Encourage students to make their own set of triangular cards for fractions, decimals and percents that they frequently use. Students can cover one or two vertices on each triangle, and practise naming the missing equivalents.

GEOMETRY AND MEASUREMENT

GEOMETRY

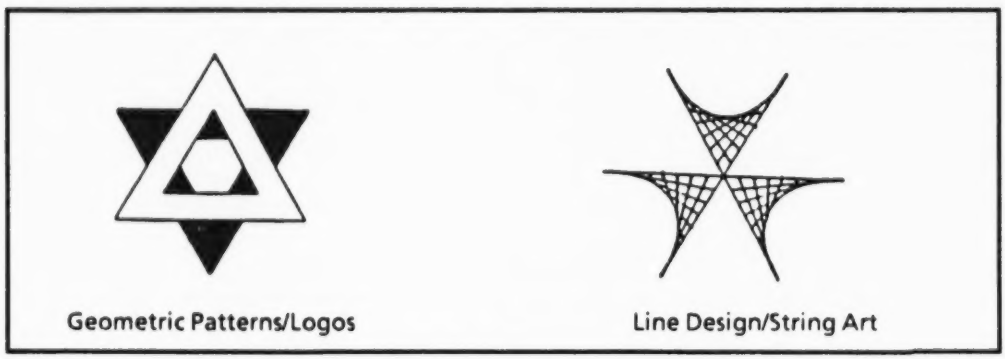
1. Students may experience difficulty in remembering the names of line relationships. Teach hand signals for horizontal, vertical, perpendicular, parallel and intersecting lines. Then play "Simon Says". Locate examples of these line relationships in the classroom.

2. Develop an understanding of the properties of basic two-dimensional shapes by:
   - identifying/discussing the properties of two-dimensional shapes present in the classroom
   - making a list of the two-dimensional shapes observed in a photograph/filmstrip/movie, or after taking a short walk
   - making a collage using various two-dimensional shapes.

3. Provide abundant opportunities for students to draw and construct the one-, two- and three-dimensional figures being studied using compass/protractor/straightedge/ruler.

   Many geometric concepts and skills can be developed/reinforced as students construct geometric logos or patterns as shown in Resource 6: Geometric Patterns. Line design and string art projects are also useful vehicles for delivering skills in geometry as found in Resource 7: Line Design.

   CLARIFICATION/EXAMPLE



Geometric Patterns/Logos

Line Design/String Art

   Encourage students to be creative in the designs and patterns they produce. Displays of student work in the classroom and hallway will enhance student motivation and effort.

   Additional ideas for geometric patterns, line design and string art projects can be obtained from a variety of resources available on these topics.

4. Use a LOGO computer program to produce geometric figures and designs. A list of LOGO commands is provided in the "Use of Technology" section of the manual. Encourage students to be creative in the designs they generate. Provide direction to students as to which line relationships/geometric figures to incorporate into the designs that are produced.

5. Ask students to create a wallpaper pattern, fabric pattern or linoleum design based on repetition of a line design/geometric pattern that has been constructed.

6. Ask students to construct the 60 attribute shapes that are required to play "Attribute Dominoes". Directions for making the shapes and playing the game are provided in Resource 8: Attribute Dominoes.

7. Provide frequent opportunities for students to solve puzzles that require the recognition of spatial patterns and relationships. Puzzles might involve experimentation with:
   - tangrams
   - tessellations
   - mazes
   - paper folding.

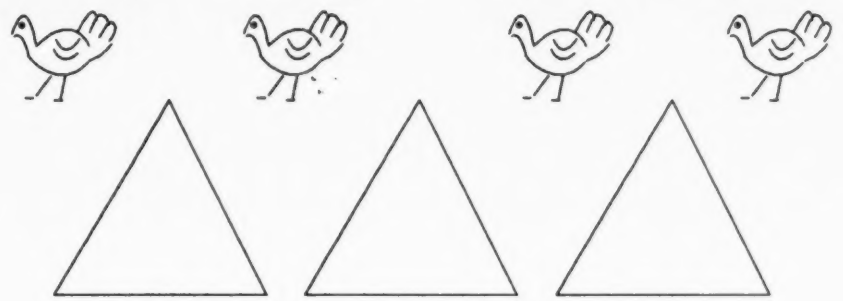
   Sample activities involving tangrams and tessellations are provided as Resource 9: The Tangram and Resource 10: Tessellations.

   A variety of books containing recreational problems and activities in mathematics can be obtained from local media centres and libraries.

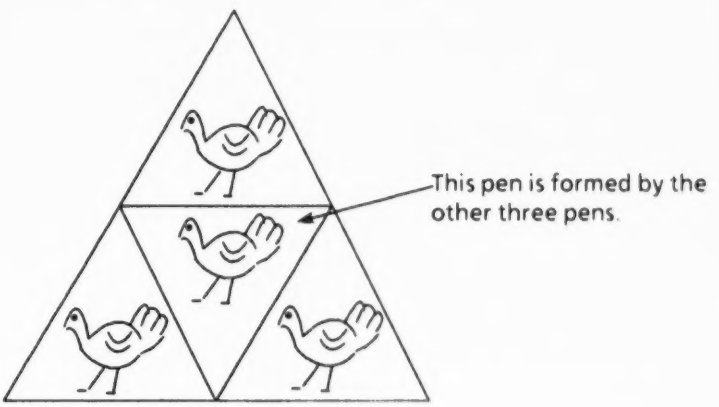
CLARIFICATION/EXAMPLE

PEN PROBLEM

A farmer has 4 turkeys, but only three triangular pens in which to cage them. How can each turkey be put in a cage of its own, using the three triangular pens?



Solution:



How many pens could be formed using 3, 4 or 5 additional triangles?

MEASUREMENT

1. Provide abundant opportunities for hands-on experience in estimating and measuring the length, mass and capacity of familiar objects.

   One method of developing familiarity with frequently used units is through the use of "mind pictures" or referents. Some examples include:
   - millimetre – thickness of a dime
   - centimetre – width of little finger
   - kilometre – distance from school to ___?___
   - gram – weight of a small paper clip
   - kilogram – weight of (subject) textbook.

   Encourage students to use these referents in making estimates, and to check their estimates through actual measurement. Coach students to select units that are appropriate to each item or object being measured. Students can record their work in tables similar to those illustrated below.

ITEM	ESTIMATED LENGTH	ACTUAL LENGTH

ITEM	ESTIMATED MASS	ACTUAL MASS

ITEM	ESTIMATED CAPACITY	ACTUAL CAPACITY

2. Another method of estimating measurements is through the use of a strategy called "chunking". The object being measured is divided into portions or chunks whose measure the student is familiar with. The measures of individual portions are then added in order to obtain an estimate of the total measurement. This strategy is particularly effective when the object being measured is large.

Example:
If you wish to measure the height of a room, divide the height into two parts:
- the height of the doorway
- the distance between the doorway and the ceiling.

3. Students may also find the strategy of "unitizing" to be an effective method of estimating the measure of a large object. Ask students to divide the object being measured into equal portions. After estimating the measure of one of the equal portions, multiply the estimate by the number of parts into which the object was divided.

4. Encourage students to recognize the importance of measurement in everyday life. Make a collage of pictures collected from newspapers and magazines that depict the use of:
   - length
   - mass
   - capacity
   - perimeter/area
   - time
   - temperature.

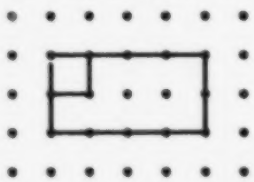
5. Plan a "scavenger hunt" that will require students to use their skills of estimation and measurement. Divide the class into small groups, and provide each group with the same list of measurements. Each group must go on a scavenger hunt and find objects that have approximately the same measurements as those on the list. No measuring devices may be used. At the conclusion of the hunt, ask each group of students to measure the objects collected. Determine the difference between the estimated and actual measurements. The team with the lowest cumulative total error is declared the winning team.

6. Develop concepts of perimeter and area through the use of tactile and manipulative materials:
   - geoboards
   - dot paper
   - tiles/tessellations.

   Compare the concept of perimeter to "distance around" and the concept of area to "surface covered".

CLARIFICATION/EXAMPLE

Form a rectangle on a geoboard or piece of dot paper. Determine its perimeter by counting units (spaces) around the outside. Determine its area by counting the square units contained within the rectangle.



Compare the area of the rectangle to its perimeter.

7. Encourage students to develop "mind pictures" for the square centimetre and square metre. Ask students to draw on paper and cut out each of these units.

   Provide opportunities for students to initially verify estimates of area they make by filling rectangular figures with tiles or square cut-outs, and counting the number of squares needed to fill the region. Once the concept of area is understood, encourage students to develop their own formulas for determining the area of familiar geometric figures.

PROJECT WORK

Project work in mathematics class often increases student motivation and provides opportunity for the application and maintenance of skills and processes. When effectively planned, a project will integrate mathematical skills with skills learned in other subject areas. Opportunity for group work will foster the development of social skills and responsibility.

Topics around which projects might be developed include:

- Planning a Vacation
- The Cost of Having a Pet
- Preparing a Personal Time Schedule
- Developing a Budget
- Designing the Ideal Bedroom
- Orienteering in the Outdoors
- Operating a Simple Retail Business
- Designing Wood/Metal/Cloth Projects

SAMPLE PROJECT: PLANNING A VACATION

Assume that a friend and yourself have decided to take a one- or two-week vacation. After selecting your destination, decide on a method of travel, plan the route, determine the costs and obtain any other information necessary to ensure an enjoyable time. Maps, brochures, pictures and other available materials should be included in your report.

OUTLINE

A. Your Destination

- What is the geographical location? Are there any time-zone changes?
- How long will your stay be?
- What will the weather be like at the time of year you will be making your trip?

B. Methods of Travel and Route

- Obtain information on all methods of travel that will be used during your vacation (e.g., how you are getting there, other travel methods you will use while there).
- Describe your route. Use maps and name any major cities through which you will be travelling.

C. Your Stay

- What kind of accommodation will you choose?
- What special attractions will you see? Include pictures or brochures if possible.
- What special things will you do there?

D. Costs

Determine the cost of your trip:

- Transportation
- Accommodation (cost per night times the number of nights)
- Approximate cost of food (cost per day times the number of days)
- Miscellaneous costs such as special tours, shows, etc.

E. Vacation Needs

- Considering the clothing and personal items you now own, prepare a list of the wardrobe, accessories and other necessities you will take on your trip.
- List additional items that you feel you must buy before you leave. Estimate their cost.
- Will you need additional medical insurance? How much will it cost?
- Will you need a passport? How do you get it?
- How much money will you need to take on your trip (e.g., cash, traveller's cheques)?

F. Additional Activities

- Calculate the total cost of the trip, based on individual costs and estimates.
- Calculate the average cost per day.
- Estimate/calculate the total distance that you will travel.
- Present your vacation plan to the class. Include pictures, posters, slides, travel brochures, maps, displays, etc.

INTEGRATED SKILLS

Skills from other subject areas that might be integrated with this project are identified below.

Mathematics:

- problem solving
- estimation
- calculator skills
- reading and interpreting the 12- and 24-hour clock
- reading and interpreting tables and charts
- determining the "best buy"
- calendar skills.

Social Studies:

- personal development and personal organization
- interpersonal skills
- decision making
- community awareness
- Canadian geography
- personal economics
- current events.

Personal and Public Services:

- personal and interpersonal skills
- body knowledge and care
- safety
- interpretation of visual data.

Business Education:

- management skills
- business ethics
- business communications
- personal skills
- interpersonal skills
- personal finances.

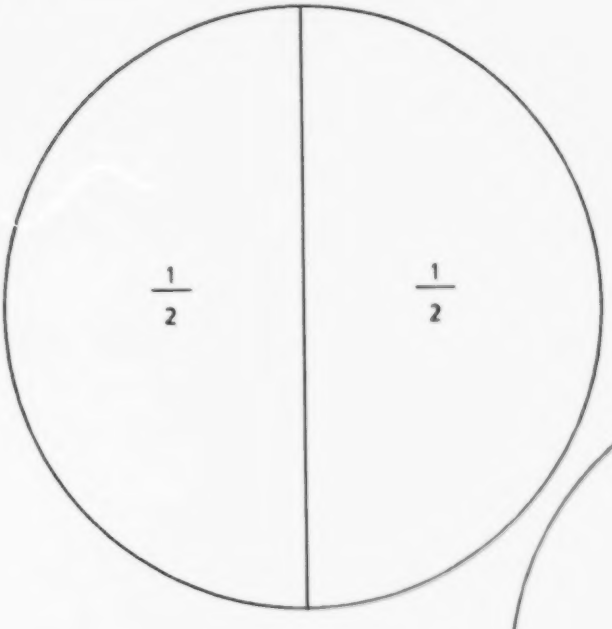
Science:

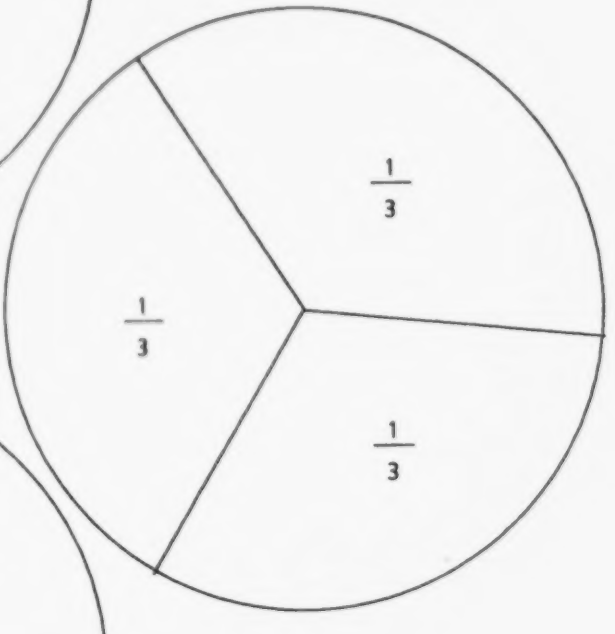
- problem solving
- decision making.

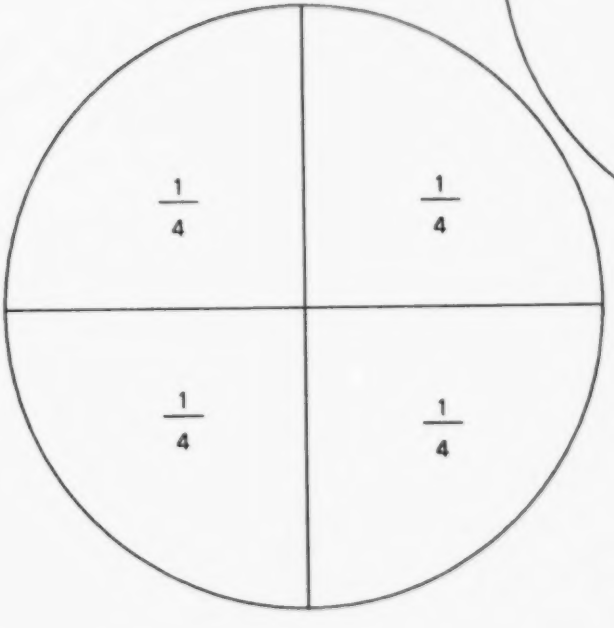
Language Arts:

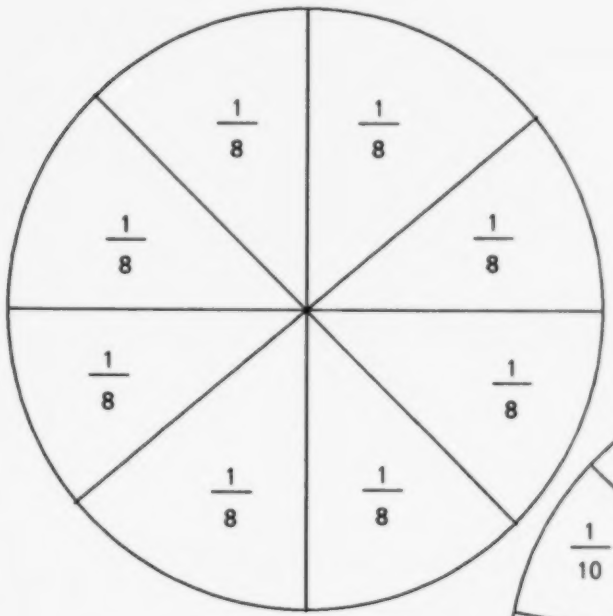
- exploring/expressing meaning
- developing interaction
- gathering information
- writing business letters.

RESOURCE 1: FRACTION CIRCLES





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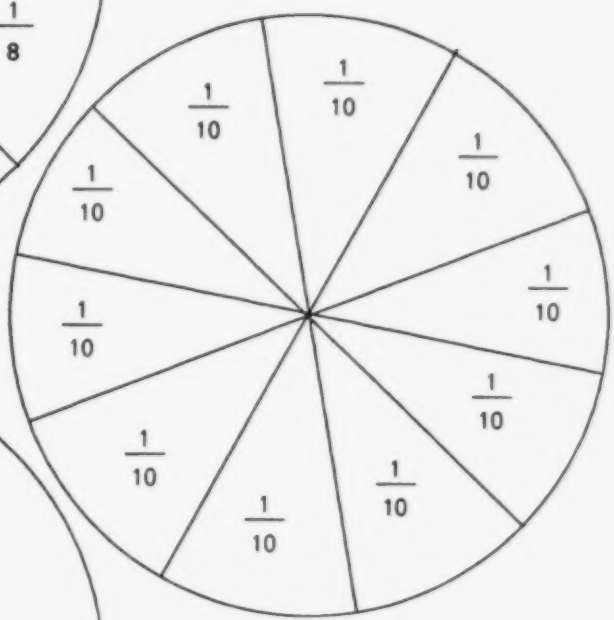
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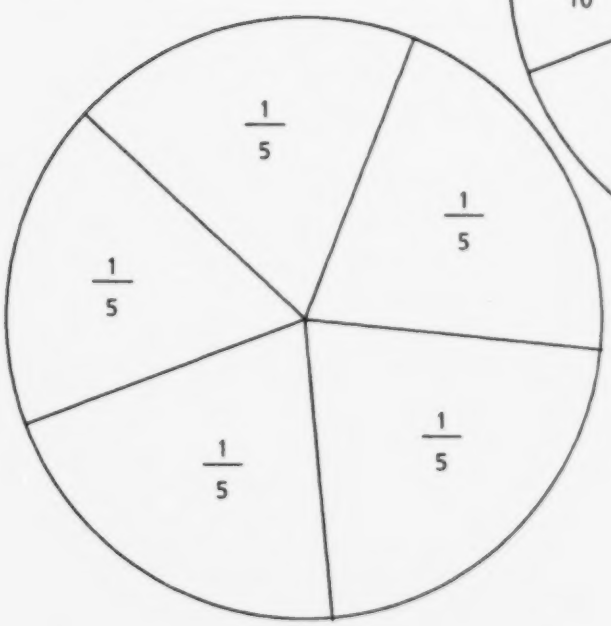
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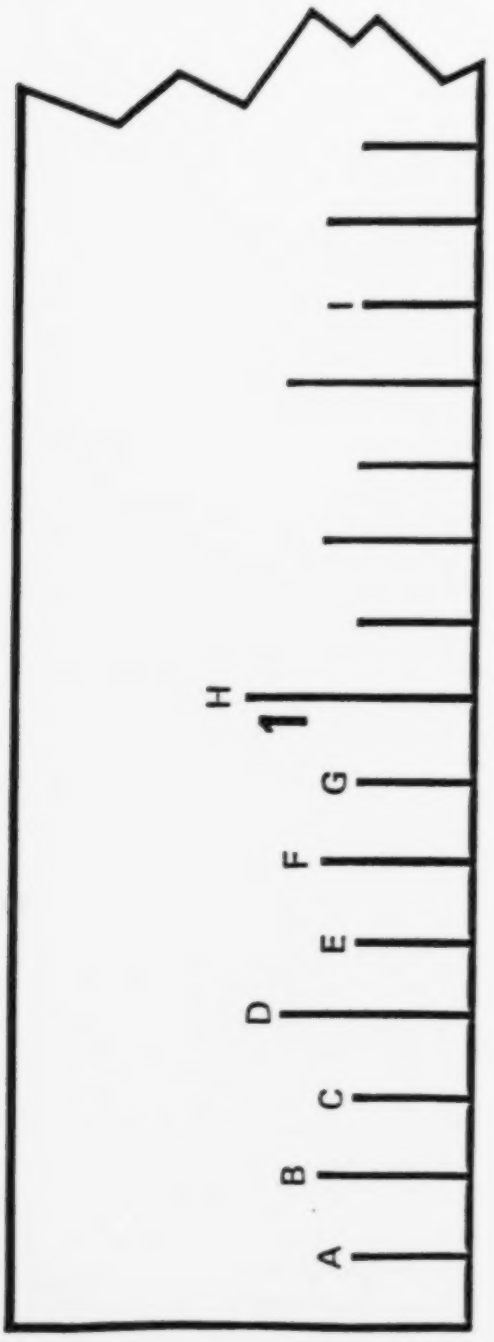


A = 1/8, B = 2/8 = 1/4, C = 3/8, D = 4/8 = 1/2,
E = 5/8, F = 6/8 = 3/4, G = 7/8, H = 8/8 = 1,
I = 13/8 = 1 5/8

...nge a Mare Lab

RESOURCE 3: INTRODUCTION TO RATIOS

MATERIALS: cuisenaire rods (one of each colour)

PROCEDURE AND OBSERVATIONS:

1. Examine the red rod. Which rod is half the length of the red rod?
   - red = _____ white
   - white = _____ red.

   The ratio of red to white is 2 to 1, which can also be written as 2:1 or as 2/1.

   The ratio of white to red is 1 to 2. Write this ratio in two other ways:
   - _____
   - _____.

2. Examine the red and light green rods. What is the relationship between the lengths of these rods?
   - red = _____ light green
   - light green = _____ red.

   The ratio of red to light green is _____ to _____. This ratio can also be written as:
   - _____
   - _____.

CONCLUSION:

Write a rule for making equal ratios.

_____

_____

_____

Test your rule. What is the ratio of light green to blue?

_____

Name two sets of coloured rods that make the same ratio:

- _____
- _____

Do these sets of coloured rods follow your rule?

_____

RESOURCE 4: EQUAL RATIOS

MATERIALS: cuisenaire rods

PROCEDURE AND OBSERVATIONS:

The ratio of purple to brown is 4 to 8, which can also be written as 4:8 or as 4/8.

Could this ratio also be written as 1 to 2? Explain.

_____

_____

_____

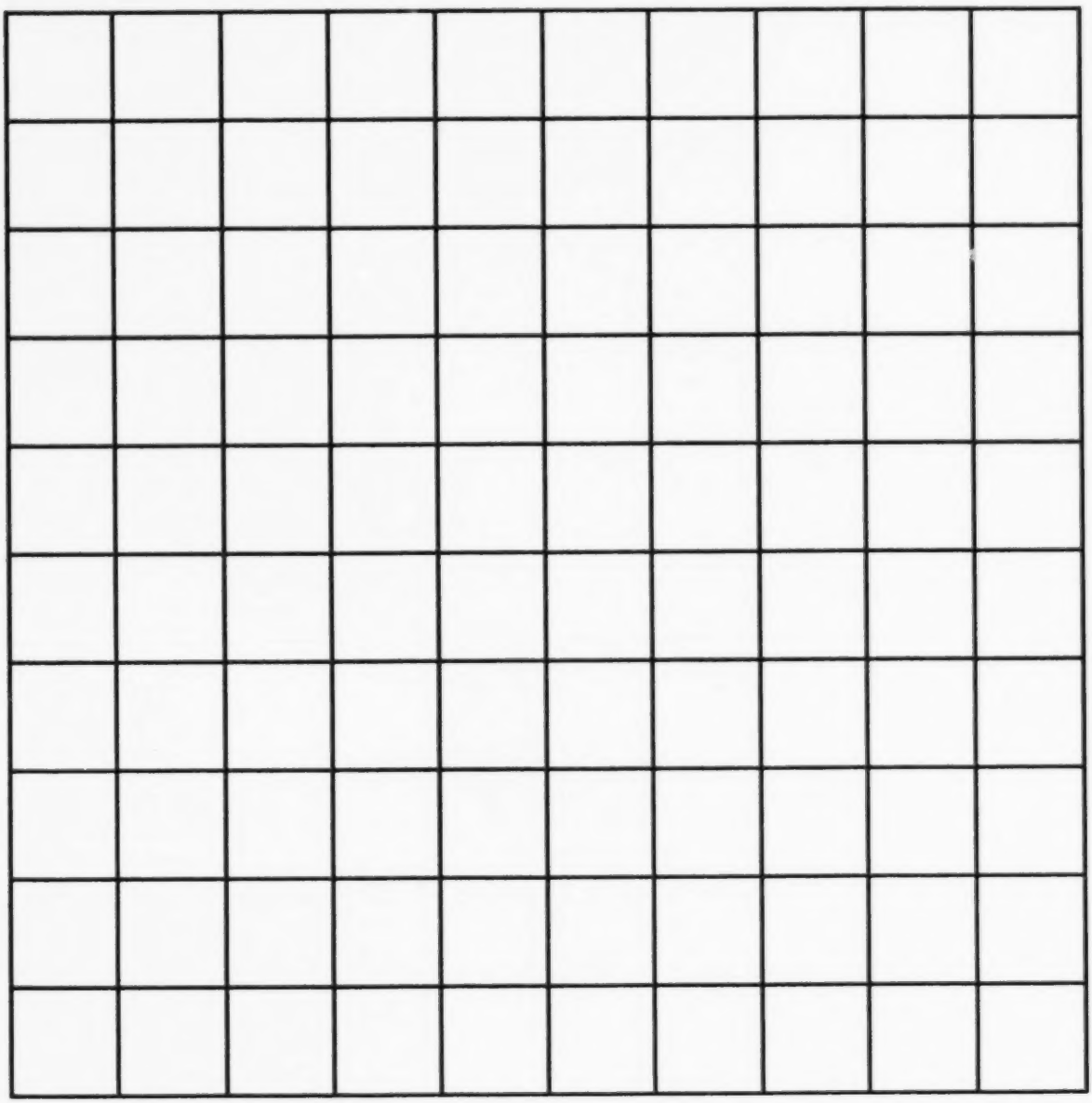
We say that 4:8 and 1:2 are equal ratios. They can be written in this way:

$\frac{4}{8} = \frac{1}{2}$.

Use your cuisenaire rods to find other ratios that are equal to 1:2. Record your results in the chart below.

COLOURS	RATIO
1. Purple to brown	4/8

RESOURCE 5: THE "100 GRID"



RESOURCE 6: GEOMETRIC PATTERNS

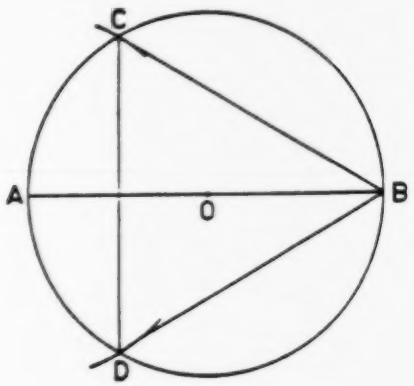
The constructions of inscribed equilateral triangles, regular hexagons, squares and regular octagons can be used to produce a variety of geometric designs.

A. TRIANGLES AND HEXAGONS

Follow the instructions for inscribing an equilateral triangle and a regular hexagon in a circle. By joining various points and colouring or shading parts of the figures, create geometric designs similar to those illustrated in the examples.

1. Inscribing an Equilateral Triangle

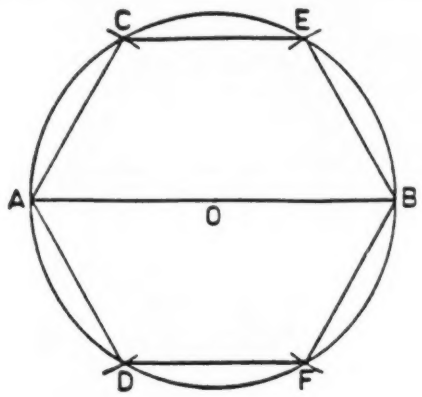
   a) With a compass draw a circle with centre 0.
   b) In this circle draw a diameter AB.
   c) With centre A and radius A0, draw an arc intersecting the circle at C and D.
   d) Connect B, C, and D to form an equilateral triangle.



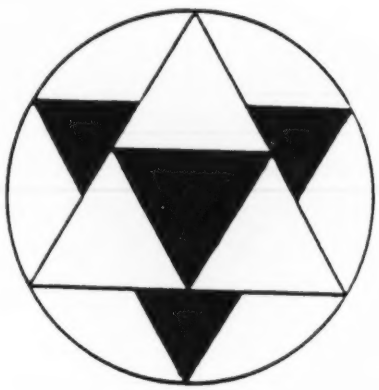
2. Inscribing a Regular Hexagon

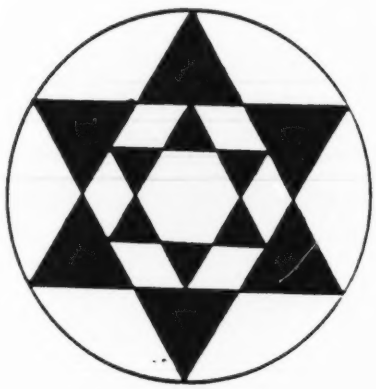
   a) With a compass draw a circle with centre 0.
   b) In this circle draw a diameter AB.
   c) With centre A and radius A0, draw arcs intersecting the circle at C and D. Similarly, with centre at B and radius B0, draw arcs intersecting the circle at E and F.
   d) Connect the six points on the circle to form a regular hexagon.

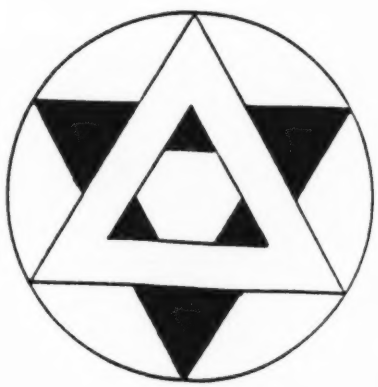
(see diagram on next page)

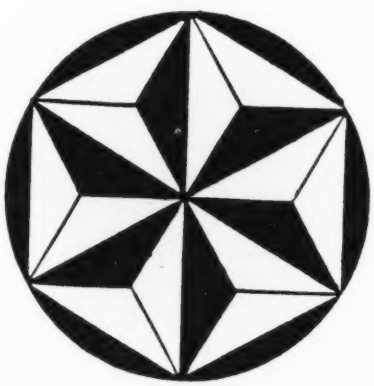


3. Examples of Designs







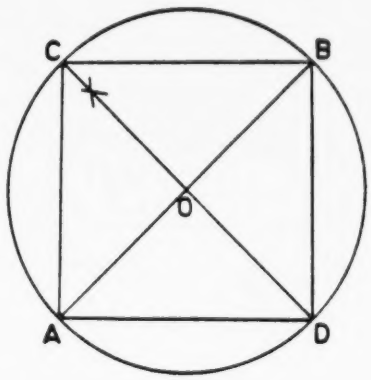


B. SQUARES AND OCTAGONS

Follow the instructions for inscribing a square and a regular octagon in a circle. By joining various points and colouring or shading parts of the figures, create geometric designs similar to those illustrated in the examples.

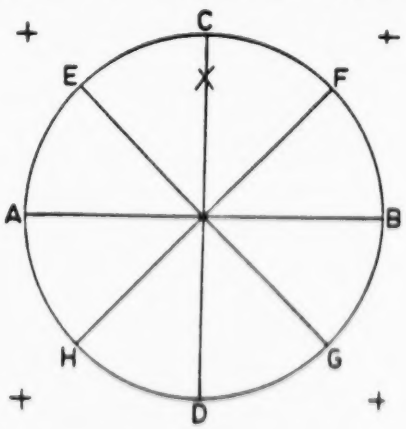
1. Inscribing a Square

   a) Draw a circle with centre 0 and draw a diameter AB.
   b) Construct another diameter CD which is the perpendicular bisector of AB.
   c) Connect the four points on the circle to form a square.

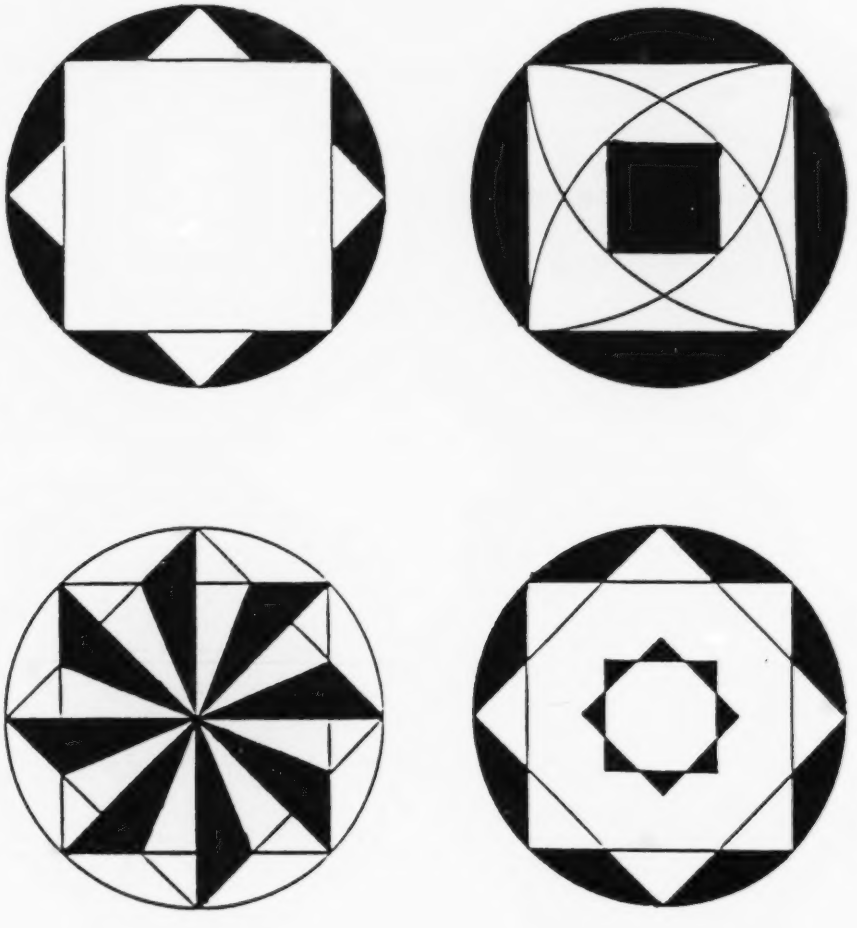


2. Inscribing a Regular Octagon

   a) Draw a circle with centre 0 and locate points A, B, C and D as in the construction of a square above.
   b) Bisect ∠AOC, ∠COB, ∠BOD, ∠DOA to obtain points E, F, G and H on the circle respectively.
   c) Connect A, E, C, F, B, G, D and H, in order, to form a regular octagon.



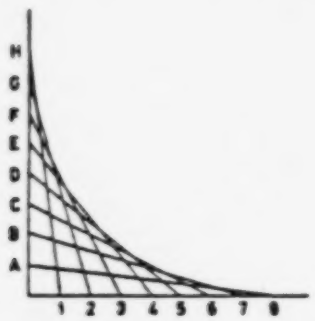
3. Examples of Designs



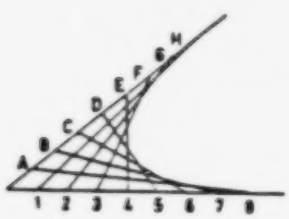
RESOURCE 7: LINE DESIGN

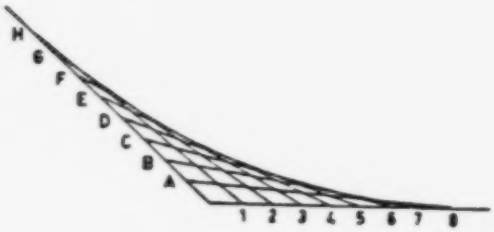
The construction of line segments can produce interesting geometric patterns and designs.

A. Draw a right angle and mark off from the vertex eight segments of equal length along each arm of the angle. (Use either a compass or a ruler to obtain equal lengths.) Label the end points of the segments as in the following diagram and connect A to 8, B to 7, . . . , and H to 1.

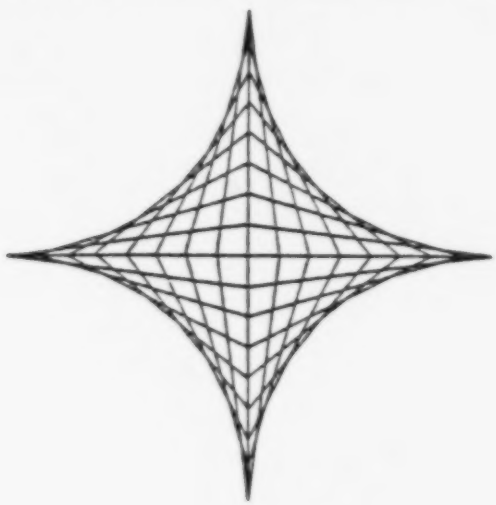


B. Try the procedure outlined above with an acute angle and an obtuse angle.

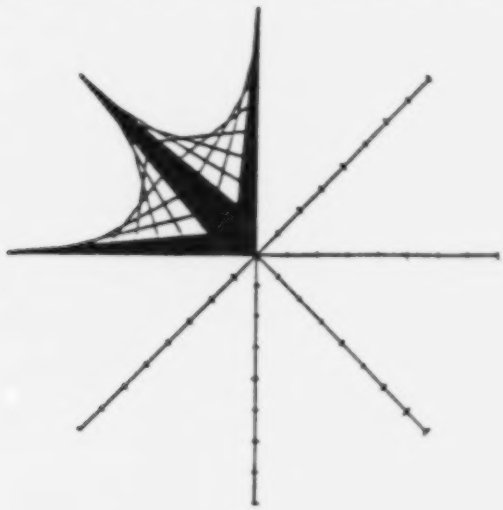




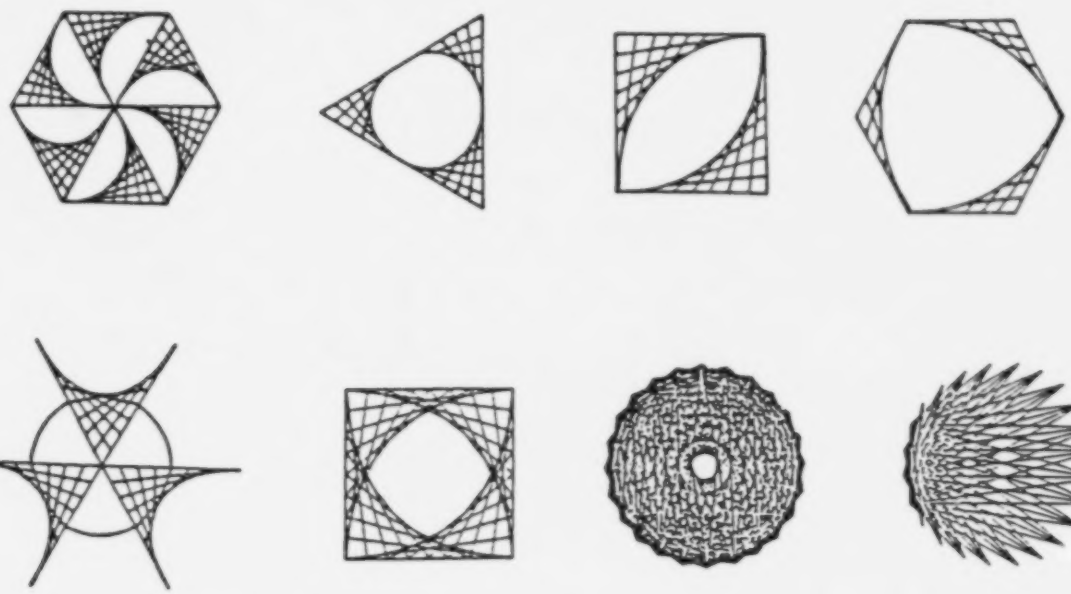
C. Draw a line segment. Construct a second line segment with the same length as the first in such a way that each segment is a perpendicular bisector of the other. Use the four angles formed to create this design.



D. More involved designs can be obtained by bisecting angles and using the angles formed for additional designs. The following example makes use of angle bisectors as well as some shading.



E. Construct additional line designs using the procedures you have learned. The illustrations below may give you some ideas.



RESOURCE 8: ATTRIBUTE DOMINOES

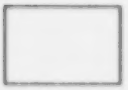
MATERIALS:

Construct a set of 60 attribute shapes using:

- three different colours of paper (e.g., blue, green, red)
- five large geometric shapes

– large square

– large circle

 – large rectangle

– large equilateral triangle

– large isosceles-right triangle

- five small geometric shapes

– small square

– small circle

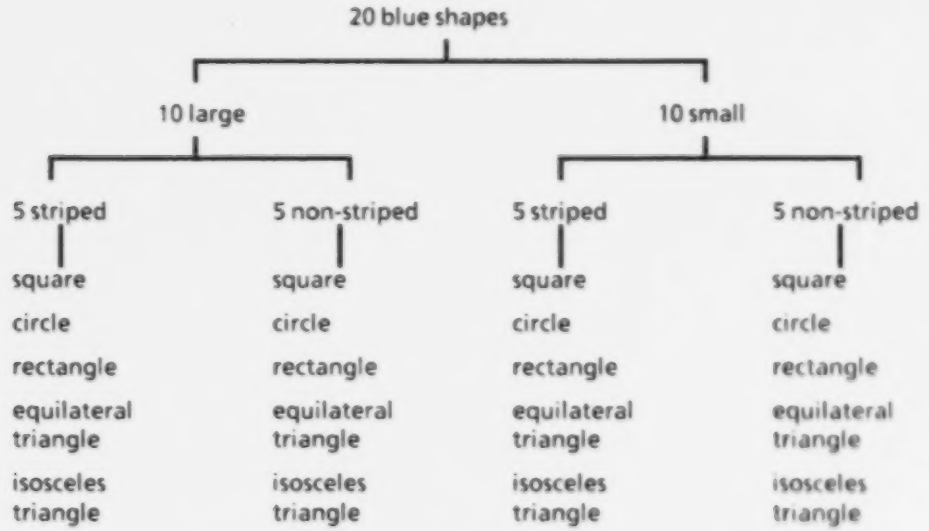
– small rectangle

– small equilateral triangle

– small isosceles right triangle

- striped and non-striped shapes in each size and colour.

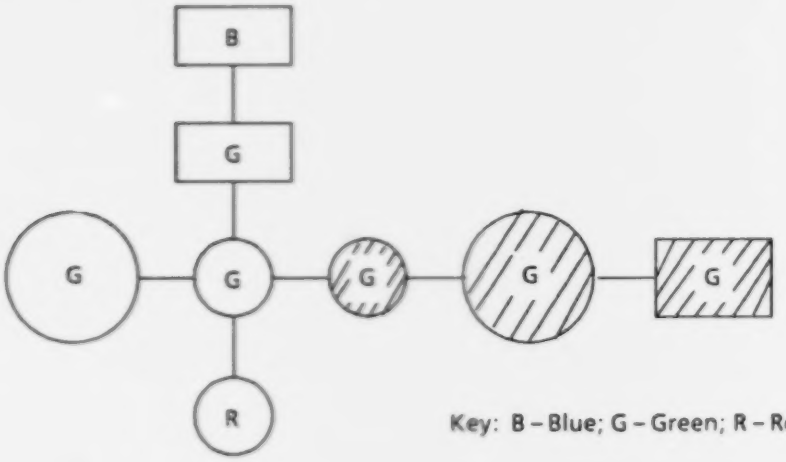
The set of 60 attribute shapes will consist of 20 blue shapes, 20 green shapes and 20 red shapes. The chart below illustrates the attributes that will be common to each set of coloured shapes.



DIRECTIONS FOR PLAY:

The game can be played with two to four players, each of whom receives ten attribute shapes. The first player places one game piece in the centre of the table. The other players take turns placing a piece in one of four directions, building from the previous piece. In order to play an attribute shape, it must differ from the piece it is placed beside by only <u>one</u> attribute. If a player cannot play, he must draw a new attribute shape from a container holding the extra pieces. If the player cannot play this shape, the play passes to the next person. The first player to play all of their attribute shapes is declared the winner.

A sample game pattern is illustrated below:



Key: B – Blue; G – Green; R – Red

NOTE: Attribute shapes can be varied according to the geometric figures that are being studied. Students in grade nine may wish to use the parallelogram, hexagon and octagon in conjunction with two other geometric shapes.

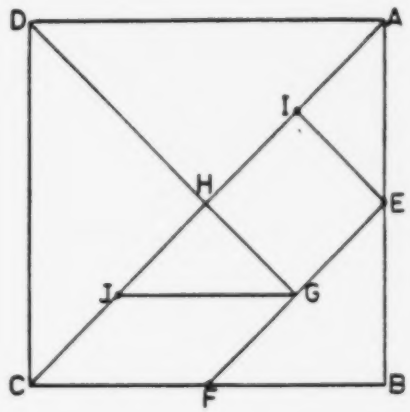
RESOURCE 9: THE TANGRAM

This exercise involves making a variety of geometric figures by rearranging pieces of a puzzle.

MAKING THE PUZZLE

If tangram pieces are not available, a set may be constructed by following these directions:

1. On cardboard draw a square ABCD with each side at least 10 cm in length.
2. Locate midpoints E and F of sides $\overline{AB}$ and $\overline{BC}$ respectively.
3. Draw $\overline{EF}$ and $\overline{AC}$.
4. Draw the portion of $\overline{DB}$ from D to $\overline{EF}$ (shown in the diagram as $\overline{DG}$). Let H be the point of intersection of $\overline{DG}$ and $\overline{AC}$.
5. Locate I, the midpoint of $\overline{AH}$, and J, the midpoint of $\overline{CH}$. Draw $\overline{EI}$ and $\overline{GJ}$.
6. Cut out the seven pieces.



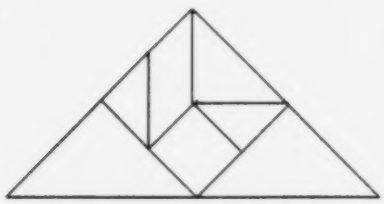
SOLVING THE PUZZLE

Using the seven tangram pieces, arrange them in such a way as to form:

1. a square
2. a triangle
3. a rectangle which is not a square
4. a parallelogram which is not a rectangle
5. a trapezoid
6. a convex polygon (a polygon in which each of the diagonals fall within its interior; 13 such polygons can be formed with the tangram pieces).

SOME POSSIBLE SOLUTIONS

2.



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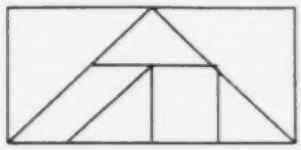

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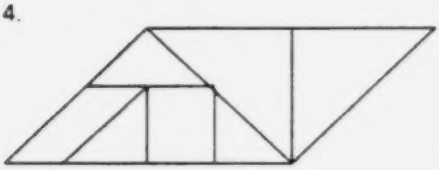

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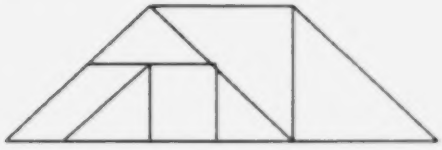
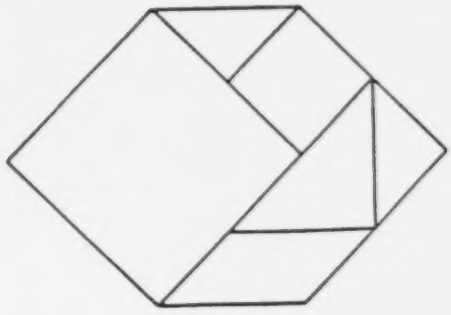


6. One convex polygon can be made in this way. See if you can make the other 12 convex polygons. Record your results.



FINDING THE AREA

Give the small square, EGHI, an area of one square unit. Compute the area of each of the other six tangram pieces using the square as the unit of measure.

RESOURCE 10: TESSELLATIONS

A. A tessellation is a pattern that can completely "fill" a surface or region without overlapping or leaving spaces. The word "tessellation" comes from an ancient Latin word which means "to cover with tiles". Some tessellations which we encounter in our everyday life are tiled floors and wallpapers. Name some other examples of tessellations.

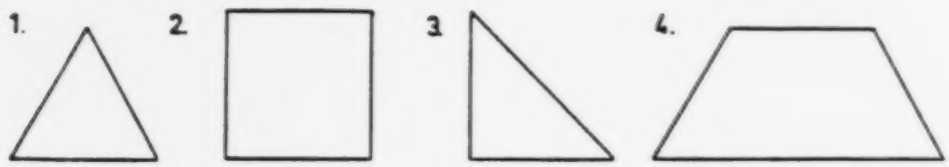
_____ _____

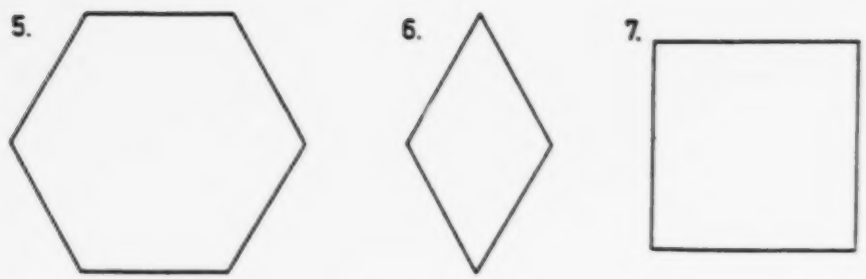
_____ _____

B. Construct tiles identical to those shown in Part C using a piece of cardboard. You will require several tiles of each shape. Your tiles will include the following shapes:

- equilateral triangle
- isosceles right triangle
- parallelogram
- rectangle
- square (three different sizes)
- rhombus
- trapezoid
- regular hexagon.

C. Using only the tiles that have the shape of an equilateral triangle or an isosceles right triangle, cover each of the following shapes. Give the name of the polygon you have covered.

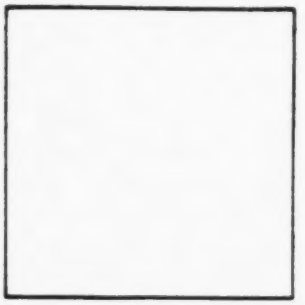




8.

9.

10.



D. Make a sketch to illustrate your use of equilateral triangles to form the hexagon in C5.

Make another sketch to show how a large equilateral triangle can be made from several small ones.

E. Can the following shapes from Point C produce tessellations?

	Yes	No
square	_____	_____
parallelogram	_____	_____
rhombus	_____	_____
trapezoid	_____	_____

Sketch the tessellations produced by each of these shapes.

In each of these tessellations, can you find a larger version of the basic shape used to make the tessellation? If you can, outline it on your sketch. Which shapes can produce tessellations but cannot be put together to make larger versions of themselves?

EVALUATION

The mathematics program emphasizes the understanding of concepts/skills, problem solving and application. Any assessment of understanding and problem solving must go beyond determining the percent of correct responses on a test based on mathematical facts. It should also provide information about how students approach doing mathematics, the level of conceptual understanding students have, and their ability to apply mathematics in new situations. The role of assessment should be to provide feedback and evidence of progress toward desired instructional goals. A singular assessment technique cannot provide this evidence.

In a report of the National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics*, the following statement is made about assessment and evaluation within the mathematics program:

> "A common form of assessment is testing for the purpose of assigning grades. But assessment should be conceived of as a much broader and basic task than just testing and grading. Its basic purpose is to determine **what** and **how** students think about mathematics. Assessment should involve the biography of students' learning as well as the continual impact of the instructional program. Such assessment should provide the basis for improving the quality of instruction. Indeed, assessment has no raison d'être unless it is clear how assessment can and will improve instruction"[1]

Furthermore, it is important to understand that through the process of evaluation we evaluate students' **performance** and not students themselves. This understanding helps one avoid permanently classifying a student as a good student or a poor student. Performance can and does change, and the teacher should be alert to significant changes.

Some evaluation strategies have been included in this section of the manual:

- Observation
- Interviews
- Inventories/Checklists
- Anecdotal Records
- Written Assignments
  - Paper-and-Pencil Tests
  - Diagnostic Writing Assignments.

These strategies are not intended to be discrete and should be used with other strategies. For example, a checklist may be used to document desirable problem-solving behaviour in the classroom, or to guide discussion and evaluate performance in an interview. The applications of these strategies should not be restricted to the suggestions provided.

[1] Reproduced with permission from *Curriculum and Evaluation Standards For School Mathematics: Working Draft*. Copyright 1987 by the National Council of Teachers of Mathematics.

OBSERVATION

Teachers observe all the time. When observations are documented, their effectiveness as an evaluation strategy increases immensely. Documented observations often provide the raw data required for analysis and diagnosis, and provide the basis on which to make remediation or enrichment decisions.

Mathematics lessons usually have a component in which students work on assignments and projects individually or in small groups. At this time teachers can observe students at work, looking for specific behaviours or outcomes, asking questions and making suggestions. Elements of the learning process that might be monitored through observation include:

- understanding of concepts/skills
- method of attacking problems
- work habits
- level of independence with work
- interpersonal skills and social growth.

Documentation of behaviours that are observed may occur in the form of anecdotal records or checklists. File anecdotal records and checklists in a student folder where samples of daily work, project reports and other artifacts are also placed.

CLARIFICATION/EXAMPLE

Anecdotal Record

Name	Date	Situation	Behaviour	Comment
John	1/17	group lesson, developing meaning of fractions with graph paper	quick to help neighbouring students	understands basic fraction concepts
	1/25	computation game	unable to use mental arithmetic when multiplying by powers of 10	redevelop and practise strategies for multiplying by powers of 10

A sample checklist that might be used in observing student behaviours has been provided as Resource 1: Observational Checklist of Student Behaviours.

Sometimes audiotapes or videotapes can be used to provide records that can be analyzed in more detail at a later time. Photographs are also useful in providing a record of project work. In addition to providing the tangible kinds of things that parents like to see at conference time, these records of student performance enable the teacher to measure progress with more objectivity than simply through memory of what was done.

INTERVIEWS

A planned interview with a student or group of students is an effective technique for assessing knowledge, understanding, thinking style, attitude or personal interests. An interview removes the restriction of writing, and enables the teacher to delve more deeply into how a student goes about finding an answer or solving a problem. Although written responses on an assignment may indicate areas of concern, more information is often required before appropriate remediation can be provided. Remediation strategies that are based solely upon the analysis of written responses may in fact be inappropriate at times. Holding interviews with students can reveal both unsuspected weaknesses and surprising strengths in their mathematical thinking.

Some guidelines for conducting interviews include:

- Establish an atmosphere of acceptance. The student must feel comfortable enough to freely verbalize his or her ideas. By accepting the student's responses without judgement, but with encouragement to elaborate further, you are communicating not only respect for the student's thinking but also a curiosity to learn more. Each response, whether right or wrong, has the potential of providing information about the student's level of understanding.
- Ask probing questions. During the interview, ask questions and introduce materials that will cause the student to extend and apply concepts/skills to new areas. Rephrase questions using vocabulary familiar to the student, so as to clarify both your intent and the student's thinking. Although some of the questions you ask should be planned, others should be invented spontaneously in order to test your hunches about the student's thinking. Questions asked may take some of the following forms:
  - How did you . . .?
  - Why did you . . .?
  - How do you know that . . .?
  - Have you . . .?
  - How did you decide whether . . .?
  - Can you describe . . .?
  - Are you sure that . . .?
  - How do you feel about . . .?
- Pace the interview appropriately. By allowing an adequate pause following each question before repeating or rephrasing it, you are giving the student time to interpret the question and construct the response. On some tasks, the student may need more than a thirty-second pause, whereas on others, three seconds will suffice. Also, by allowing an adequate pause following the student's response, you are indirectly encouraging him or her to elaborate on his or her initial response.
- Be prepared to coax and encourage the student to make a response. A frequent response given by the student may be "I don't know" or "I forgot". Sometimes an extended pause can coax out productive thinking. At other times, the teacher can encourage a response to questions about which the student is unsure by saying:
  - "I know it's not easy to think about. Just give me your best idea."
  - "Pretend that you did know. How do you think it might be done?"

Interviews should have a definite purpose and both teacher and student should be aware of the purpose. Interviews must be planned in advance. In preparing for an interview, the teacher should consider:

- What questions will I ask?
- What basic understandings will I initially assess?
- How can I incorporate the use of manipulative materials?
- How can I vary the task and questions to obtain different perspectives on the student's ability?

Maximum benefits can be gained from an interview by reflecting on your interaction after listening to an audiotape playback. Considerable value can also be gained from sharing your tape with a colleague. By discovering how students interpret and view a problem, the teacher will be better able to make effective on-the-spot decisions in the classroom.

Several guides for interviewing have been included at the end of this section of the manual:

- Resource 2: Sample Protocol for Student Interviews
- Resource 3: Interview Guide for Problem Solving
- Resource 4: Interview Guide for Project Work.

INVENTORIES/CHECKLISTS

Inventories and checklists are documentation strategies that are used conjunctively with other evaluation strategies. They can be easily designed and customized to meet many different needs and situations. Generally a matrix is created, with indicators of desirable behaviours/outcomes on one side, and ratings/skill levels along another side. As teachers note a particular behaviour, they need only check the appropriate column that evaluates or rates the behaviour.

Checklists lend themselves very well to documenting such elements of the program as:

- comprehension of a concept when using a manipulative
- mastered knowledge, skills or process
- work habits
- problem-solving strategies
- social skills.

CLARIFICATION/EXAMPLE

Checklist/Inventory of Social Skills

Behaviour to be Observed	Frequently	Sometimes	Never
Is sensitive to the needs and problems of others.			
Willingly shares ideas/materials.			
Accepts suggestions and help.			
Adheres to group plans/decisions.			
Works cooperatively with others.			
Respects the property of others.			
Appears to like group work.			

A variety of inventories/checklists useful in monitoring the development of problem-solving skills are provided in the "Problem Solving" section of this manual:

- Attitude Inventory Items
- Observational Checklist of Problem-Solving Attitudes and Behaviours
- Observational Rating Scale of Problem-Solving Attitudes and Behaviours
- Checklist of Problem-Solving Strategies
- Problem-Solving Strategy Inventory.

Similar checklists can be developed to monitor and evaluate other components of the mathematics program.

e.g., – use of estimation strategies
– facility with mental arithmetic.

ANECDOTAL RECORDS

Anecdotal records refer to the spontaneous documentation of notable behaviour, effort and achievement. These records provide specific and dated information that can form the basis for conclusions and assessments. Anecdotal records often prove invaluable in clarifying assessments and add credibility to observations and recommendations being offered in student, parent and/or teacher meetings.

Anecdotal records may include observations on:

- attitude/work habits
- social skills
- effort and homework
- changes in performance
- specific strengths/deficiencies
- communication skills.

CLARIFICATION/EXAMPLE

Anecdotal Record Card

Student: Sue Jones Date: 04/10

Comments:

- knows how and when to look for a pattern in problem solving
- keeps trying even when she has trouble finding a solution
- shows interest in helping others
- often enters numbers into the calculator in the wrong order when dividing

Anecdotal records may be kept in a daily or weekly diary, in student files, in the marks record book, or in a common file of short, dated notes.

WRITTEN ASSIGNMENTS

PAPER-AND-PENCIL TESTS

Traditional paper-and-pencil tests are probably the most widely used method of evaluating student performance. While effective in assessing factual and procedural knowledge, these tests often elicit feelings of inadequacy and self-doubt for students who have experienced previous difficulty or failure. These negative feelings affect test performance, and may cause a cycle of repeated failure to continue.

Students may have learned the information presented in class, but are unable to demonstrate the knowledge because of poor reading skills, visual perception problems, inadequate reasoning and comprehension, fine-motor difficulties or other related deficiencies. Special needs of the student can be met through minor alterations in the construction of teacher-made tests. Constructing tests according to special needs can mean the difference between success and failure for some students.

The suggestions which follow will assist teachers to construct their tests according to the needs of individual students.

TEST DIRECTIONS

- Keep directions simple and avoid unnecessary words.
- Define words that are unfamiliar or abstract.
- Give an example of how the student is to respond.

CLARIFICATION/EXAMPLE

Directions

Add the fractions.
Give the answer in lowest terms.

Example:

$$\begin{array}{r} \frac{1}{3} = \frac{2}{6} \\ + \\ \frac{1}{2} = \frac{3}{6} \\ \hline \frac{5}{6} \end{array}$$

Some teachers feel that providing a model invalidates a test designed to measure knowledge of mathematical process. However, some students may never be able to remember a formula or complex set of processes without visual prompts. For them, failure is almost a certainty without modifications.

- Avoid oral directions as the only means of communication. Read directions orally as well as clearly writing them on the test.

TEST ITEMS

- Provide manipulative objects that make the problems more concrete.
- When using computation problems, avoid mixing different problem formats in the same section.

CLARIFICATION/EXAMPLE

A student with organizational or visual tracking problems may have difficulty in "changing gears" from problem A to problem B. It might be better to put these questions in two different sections of the test.

Problem A	Problem B
468 + 31 896	670 + 40 + 861 =

- Provide visual prompts for computational problems.

CLARIFICATION/EXAMPLE

□ 73
+ 69

OR

$$61\overline{)263}\ \text{R}$$

- Provide formulas and explain the meaning of special symbols (e.g., A = l x w, < means "less than").
- Provide a set of written steps for applying difficult algorithms.

CLARIFICATION/EXAMPLE

Long Division

1. Divide
2. Multiply
3. Subtract
4. Check
5. Bring down

- Incorporate the use of the calculator into computational problems that are not intended to assess understanding of paper-and-pencil process.
- Design word problems that:
  - are relevant to the student's personal experience
  - consist of simple sentences and familiar words.
- Underline or circle key words in word problems (e.g., less, (more)).
- Ask students to circle the correct response in multiple choice items. This reduces the possibility of copying errors when transferring letters to blanks. Arrange the answer and distractors vertically on the page.

CLARIFICATION/EXAMPLE

You have a board 48 centimetres long. If you cut off a 6-centimetre piece, how much is left?

a. 38 centimetres
b. 42 centimetres
c. 48 centimetres

- Keep all matching items brief, and have only one correct answer for each item. Use no more than ten items in the matching lists. If you have more than ten items, group them by concepts in clusters of ten.

TEST DESIGN

- Construct the test in logical sequential order, from simple to complex problems.
- Use test items that reflect the content taught and techniques used to teach.
- Prepare a study guide for the test that matches the design of the actual test.
- Design the test to reflect the student's knowledge, rather than ability to follow complicated directions, to use difficult vocabulary, or to work under time constraints.
- Adjust the readability level of the test to meet student needs.
- Prepare the test in short sections that can be administered individually if necessary.
- Use graph paper for paper-and-pencil computational problems. The squares may help the student to keep figures aligned.

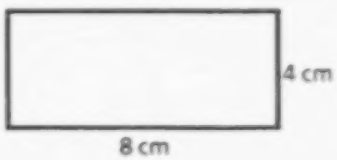
DIAGNOSTIC WRITING ASSIGNMENTS

Diagnostic writing assignments require the student to respond to specific mathematical questions in an expressive writing style. Written responses often force the student to examine their own understanding of concepts and will communicate to teachers how much students really know about a concept. Written responses also provide insight into how the student thinks. These assignments have proven successful as a diagnostic tool. Appropriate remedial and enrichment activities may be determined on the basis of the understanding demonstrated for a given concept.

CLARIFICATION/EXAMPLE

Comparing Perimeter and Area

What is wrong with each of the following statements about the rectangle?



- The area is 32 cm.
- The perimeter is 12 cm.
- The area is 12 cm^2.
- If the length is increased by 2 cm, the area will be increased by 2 cm^2.
- If the perimeter is doubled, the area will be doubled.
- The perimeter of any rectangle is always smaller than the area.

Written assignments at this grade level should be short. They should not be graded for a mark but rather, assessed for understanding (diagnosis). Students might be awarded bonus points based on their effort and presentation.

CLARIFICATION/EXAMPLE

0 points	–	no effort
1 point	–	mediocre attempt with little or no understanding
2 points	–	good attempt, but with some lack of understanding
3 points	–	high level of effort and understanding

Diagnostic writing assignments can be kept in a student diary or logbook, and might be assigned on a regular basis (once or twice a week), or in lieu of a regular quiz.

RESOURCE I: OBSERVATIONAL CHECK LIST OF STUDENT BEHAVIOURS

LEVEL OF INDEPENDENCE WITH WORK:	ALWAYS	OFTEN	SOMETIMES	NEVER
• Settles down to work upon entering class.				
• Spends time on task; has satisfactory attention span.				
• Takes responsibility for making up work after absences.				
• Takes responsibility for supplies and equipment.				
• Follows directions; completes tasks with minimal assistance.				
• Asks for and accepts help when needed.				
• Accepts a challenge; works productively on tasks of increasing difficulty.				
• Displays self-confidence and pride in work.				

UNDERSTANDING OF CONCEPTS AND SKILLS:				
• Uses the necessary vocabulary and concepts.				
• Uses appropriate operations, strategies and principles.				
• Asks questions, volunteers answers.				
• Answers questions that involve thought (e.g., What do you think?).				
• Demonstrates understanding through ability to generalize and apply.				
• Displays curiosity about concepts, relationships and applications.				
• Works independently on projects and research.				

ABILITY TO INVESTIGATE AND SOLVE PROBLEMS:

	ALWAYS	OFTEN	SOMETIMES	NEVER
• Understands and defines problems.				
• Develops a systematic plan of attack.				
• Gathers information using a variety of sources.				
• Carries out plans and procedures, seeking help when necessary.				
• Uses appropriate strategies and processes.				
• Considers alternatives before reaching a solution/decision.				
• Evaluates solutions to the problem and decisions made.				
• Considers other ideas/opinions/solutions.				

INTERPERSONAL SKILLS AND SOCIAL GROWTH :

• Resists aggressive and impulsive behaviours.				
• Volunteers to work in group situation.				
• Cooperates and contributes to group goals.				
• Listens to peers; considers the opinions of others.				
• Participates in oral discussions.				
• Helps others willingly.				

RESOURCE 2: SAMPLE PROTOCOL FOR STUDENT INTERVIEWS[1]

Sample interview questions that might be used in assessing the student's understanding of operations, basic facts and algorithmic procedures are provided. It is not suggested that every question be used during each interview. Their purpose is simply to illustrate how carefully sequenced and well-phrased questions can provide worthwhile information about the student's understanding of process and skill.

UNDERSTANDING OF OPERATIONS

The following questions and tasks relate to the operation of division. Similar questions and tasks could be developed for other operations. These questions will indicate:

- how students interpret the operation
- whether the action can be identified and simulated
- whether the operation can be related to personal experience.

INTERVIEW QUESTIONS:

1. Read "56 ÷ 7" for me.

2. Do you know other ways of reading this? Students may frequently read statements like 56 ÷ 7 or equivalent subtraction statements (e.g., 56 – 7) in either direction.

3. Do you know other names for " ÷ "? (Point to the symbol, or use terminology from the student's response to the previous task.)

4. Use counters (or draw a sketch) to show 12 ÷ 3.
   - Is the student able to simulate the action for 12 ÷ 3 with counters?
   - Is the student's preferred division interpretation one of measurement or partition, and will the student be consistent throughout the interview?

5. Make up a word problem for 12 ÷ 3.

KNOWLEDGE OF BASIC FACTS

The following examples deal with multiplication facts. Similar strategies could be developed for basic facts related to other operations. Through appropriate interview questions, the teacher can assess the student's ability to:

- recall simple basic facts
- use known facts to derive answers for other facts
- apply the properties of "zero" and "one"
- use strategies for checking answers or finding an answer for a fact that is not known.

[1] Reprinted with permission from *The Arithmetic Teacher*. Copyright 1988 by the National Council of Teachers of Mathematics.

INTERVIEW QUESTIONS:

1. Give the answer for 4×0.

2. What is your rule for finding the answer when a number is multiplied by zero? Use your rule for 25×0.

3. Give the answer for 6×1.

4. What is your rule for finding the answer when a number is multiplied by one? Use your rule for 37×1.

5. If the answer for 7×9 is 63, how could you use this answer to find the answer for 8×9?

6. Pretend you forgot the answer for 7×8. How would you find or calculate this answer?

7. As I show you a flash card, tell me, without calculating the answer, whether you think the fact is easy or hard. (After the cards have been sorted as "easy" and "hard", answers are solicited and checked by turning the appropriate cards over, first for the facts classified as easy and then for those identified as hard.)

UNDERSTANDING OF ALGORITHMIC PROCEDURES

Subtraction is used to illustrate a possible protocol for an algorithmic procedure during an interview. These procedures can be adapted for other operations. Questions posed during the interview should relate to:

- the order of "attack" used by the student
- the reasons for "moves" made during the calculation
- the meaning of the digits that are being manipulated.

INTERVIEW QUESTIONS:

1. Show me how you would find the answer for 57 – 34 = _____. Talk to me as you are doing it.

   Explanations provided by the student should provide answers for the following questions:
   - Why did you record one number below the other?
   - Why did you begin "here"?
   - You said "5 – 3". What do these digits really mean?

2. Explain to me what you are thinking as you find the answer for 71 – 48 = _____.

3. Show me how you would explain how to find the answer for 4003 – 897 = _____ to a student in a lower grade (or to a younger brother or sister).

4. Use these base-ten blocks to show how you would find the answer for 605 – 67 = _____.

RESOURCE 3: INTERVIEW GUIDE FOR PROBLEM SOLVING[1]

1. Establish rapport to help the student feel comfortable.

2. Ask the student to "talk about what he or she is doing or thinking" while solving the problem. Explain that this will enable you to help the student to become a better problem solver.

3. Give this problem to the student:

   At an amusement park, Mike and his 5 friends decided to take enough roller coaster rides so that each person would take a ride with every other person exactly once. How many rides were taken if only 2 students went on each ride?

4. As the student attempts to understand the problem question and conditions, observe the student and ask questions such as the following, if appropriate:

   a. What did you do first when given the problem? Next?
   b. What question is asked in the problem? What are the important facts and conditions in the problem? Do you need any information not given in the problem?
   c. Is there anything you don't understand about the problem?

5. As the student works on a solution to the problem, remind him or her again to talk about it, and ask questions such as the following, if appropriate:

   a. What strategy are you using? Do you think it will lead to a solution? Have you thought about using other strategies? Which ones?
   b. Where are you having difficulty? What are your ideas about where to go from here?

6. As the student finds an answer to the problem, observe the ways, if any, in which he or she checks the answer and its reasonableness as a solution. Ask questions such as:

   a. Are you sure this is the correct answer to the problem? Why?
   b. Do you think it is important to check your answer? Why?

7. After the student has solved the problem, ask questions such as:

   a. Can you describe the solution to the problem and how you found it?
   b. Is this problem like any other problem you've solved? How?
   c. Do you think this problem could be solved in another way? What are your ideas?
   d. How did you feel while you were solving this problem? How do you feel now that you have found a solution?

[1] Reprinted with permission from *How to Evaluate Progress in Problem Solving*. Copyright 1987 by the National Council of Teachers of Mathematics.

RESOURCE 4: INTERVIEW GUIDE FOR PROJECT WORK[1]

THE TASK ENVIRONMENT

1. How did you select this investigation/project? _____

   _____

2. Are you interested in it?
   _____ very _____ somewhat _____ not at all

3. What did the teacher do when giving out the assignment? _____

   _____

   Examples:
   - give verbal guidelines
   - give written guidelines
   - select the topic
   - provide a strategy
   - increase your interest

4. Who do you expect will examine the results of your investigation/project? _____

   _____

PREVIOUS KNOWLEDGE

5. Have you undertaken an investigation/project like this before?
   _____ yes _____ no

6. What did you know about this topic before you started? _____

   _____

7. Which of the following mathematical ideas/processes did you use in carrying out your investigation/project?

_____ geometric shapes	_____ large numbers
_____ order of operations	_____ decimals
_____ fractions	_____ estimation
_____ rounding numbers	_____ calculator usage
_____ mental arithmetic	_____ graphs and tables
_____ units of measure	_____ budgeting
_____ banking skills	_____ comparative shopping
_____ scale drawing	_____ patterns and designs

[1] For permission to reprint copyrighted material, grateful acknowledgement is made to the following: Division for Learning Disabilities, *Learning Disabilities Focus* for the material adapted from "Alternatives in the Assessment of the Learning Disabled Adolescent: A Learning Strategies Approach" by Judith Wiener, Spring 1986.

PLANNING

8. How did you plan your investigation/project? _____

_____

9. Did you make an outline of the steps you would take?
_____ yes _____ no

What kind of thinking did you do first? _____

_____

10. Did you do any research? _____ yes _____ no

What sources of information did you use? _____

_____

REVIEWING AND APPLYING

11. What strategies did you find most helpful in completing your investigation/project?

Examples:

_____ reading and research
_____ following directions received
_____ explanations from the teacher
_____ studying diagrams/models
_____ receiving help from parents
_____ studying with a partner
_____ explaining problems to classmates
_____ having group discussions about solutions
_____ other

12. Explain three ways that you might make use of the results of your investigation in everyday life.

_____

_____

_____

EVALUATING

13. What have you learned in completing this investigation/project? _____

_____

14. What grade did you think you would get? _____

Why? _____

15. What was the teacher's evaluation? _____

How was this evaluation different from your anticipated grade? _____

_____

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